

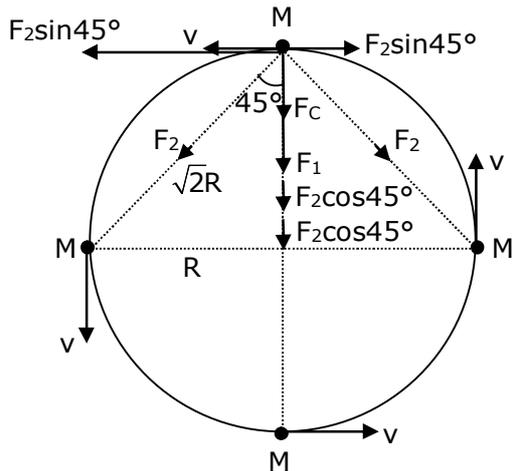
# 24<sup>th</sup> Feb. 2021 | Shift - 1 PHYSICS

## Section - A

1. Four identical particles of equal masses 1 kg made to move along the circumference of a circle of radius 1 m under the action of their own mutual gravitational attraction. The speed of each particle will be -

(1)  $\frac{\sqrt{(1+2\sqrt{2})G}}{2}$       (2)  $\sqrt{G(1+2\sqrt{2})}$       (3)  $\sqrt{\frac{G}{2}(2\sqrt{2}-1)}$       (4)  $\sqrt{\frac{G}{2}(1+2\sqrt{2})}$

Sol. (1)



⇒ By resolving force  $F_2$ , we get

$$\Rightarrow F_1 + F_2 \cos 45^\circ + F_2 \cos 45^\circ$$

$$\Rightarrow F_1 + 2F_2 \cos 45^\circ = F_c$$

$$F_c = \text{centripital force} = \frac{MV^2}{R}$$

$$\Rightarrow \frac{GM^2}{(2R)^2} + \left[ \frac{2GM^2}{(\sqrt{2}R)^2} \cos 45^\circ \right] = \frac{MV^2}{R}$$

$$\Rightarrow \frac{GM^2}{4R^2} + \frac{2GM^2}{2\sqrt{2}R^2} = \frac{MV^2}{R}$$

$$\Rightarrow \frac{GM}{4R} + \frac{GM}{\sqrt{2}R} = V^2$$

$$\Rightarrow V = \sqrt{\frac{GM}{4R} + \frac{GM}{\sqrt{2}R}}$$

$$\Rightarrow V = \sqrt{\frac{GM}{R} \left[ \frac{1+2\sqrt{2}}{4} \right]}$$

$$\Rightarrow V = \frac{1}{2} \sqrt{\frac{GM}{R} (1+2\sqrt{2})}$$

(given : mass = 1 kg, radius = 1 m)

$$\Rightarrow v = \frac{1}{2} \sqrt{G(1+2\sqrt{2})}$$

2. Consider two satellites  $S_1$  and  $S_2$  with periods of revolution 1 hr. and 8 hr. respectively revolving around a planet in circular orbits. The ratio of angular velocity of satellite  $S_1$  to the angular velocity of satellite  $S_2$  is -

- (1) 8 : 1                      (2) 1 : 8                      (3) 2 : 1                      (4) 1 : 4

Sol. (1)

We know that  $\omega = \frac{2\pi}{T}$

given : Ratio of time period

$$\frac{T_1}{T_2} = \frac{1}{8}$$

$$\Rightarrow \omega \propto \frac{1}{T}$$

$$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$$

$$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{8}{1}$$

$$\Rightarrow \omega_1 : \omega_2 = 8 : 1$$

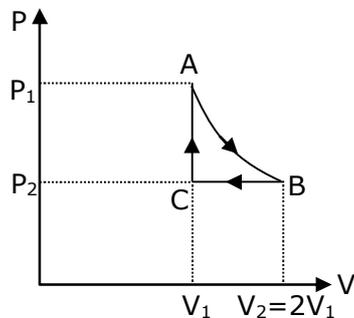
3. n mole of a perfect gas undergoes a cyclic process ABCA (see figure) consisting of the following processes -

A → B : Isothermal expansion at temperature T so that the volume is doubled from  $V_1$  to  $V_2 = 2V_1$  and pressure changes from  $P_1$  to  $P_2$ .

B → C : Isobaric compression at pressure  $P_2$  to initial volume  $V_1$ .

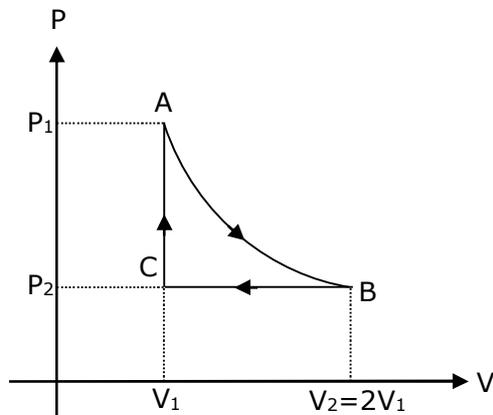
C → A : Isochoric change leading to change of pressure from  $P_2$  to  $P_1$ .

Total workdone in the complete cycle ABCA is -



- (1) 0                      (2)  $nRT \left( \ln 2 + \frac{1}{2} \right)$                       (3)  $nRT \ln 2$                       (4)  $nRT \left( \ln 2 - \frac{1}{2} \right)$

**Sol. (4)**



A → B = isothermal process

B → C = isobaric process

C → A = isochoric process

also,  $V_2 = 2V_1$

work done by gas in the complete cycle ABCA is -

$$\Rightarrow W = W_{AB} + W_{BC} + W_{CA} \quad \dots(1)$$

$$\Rightarrow W_{CA} = 0, \text{ as isochoric process}$$

$$\Rightarrow W_{AB} = 2P_1V_1 \ln \left( \frac{V_2}{V_1} \right) = 2nRT \ln(2)$$

$$\Rightarrow W_{BC} = P_2(V_1 - V_2) = P_2(V_1 - 2V_1) = -P_2V_1 = -nRT$$

⇒ Now put the value of  $W_{AB}$ ,  $W_{BC}$  and  $W_{CA}$  in equation, we get

$$\Rightarrow W = 2nRT \ln(2) - nRT + 0$$

$$\Rightarrow W = nRT [2 \ln(2) - 1]$$

$$\Rightarrow W = nRT \left[ \ln(2) - \frac{1}{2} \right]$$

**4.** Two equal capacitors are first connected in series and then in parallel. The ratio of the equivalent capacities in the two cases will be -

(1) 2 : 1

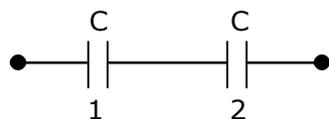
(2) 1 : 4

(3) 4 : 1

(4) 1 : 2

**Sol. (2)**

Given that first connection



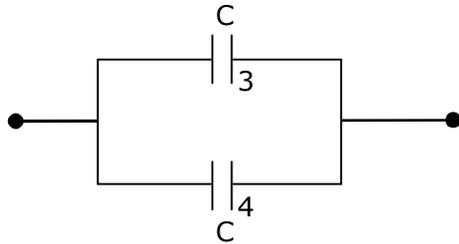
$$\Rightarrow \frac{1}{C_{12}} = \frac{1}{C} + \frac{1}{C} \Rightarrow C_{12} = \frac{C}{2}$$

Second connection

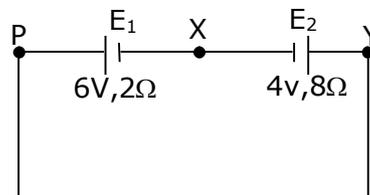
$$C_{34} = C + C = 2C$$

Now, the ratio of equivalent capacities in the two cases will be -

$$\Rightarrow \frac{C_{12}}{C_{34}} = \frac{C/2}{2C} \Rightarrow \frac{C_{12}}{C_{34}} = \frac{1}{4}$$

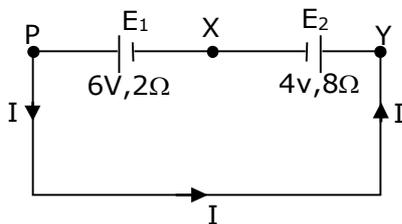


5. A cell  $E_1$  of emf 6V and internal resistance  $2\Omega$  is connected with another cell  $E_2$  of emf 4V and internal resistance  $8\Omega$  (as shown in the figure). The potential difference across points X and Y is -



- (1) 3.6V                      (2) 10.0V                      (3) 5.6V                      (4) 2.0V

**Sol. (3)**



emf of  $E_1 = 6\text{v}$

$r_1 = 2\ \Omega$

emf of  $E_2 = 4\ \Omega$

$r_2 = 8\ \Omega$

$|v_x - v_y| =$  potential difference across points x and y

$E_{\text{eff}} = 6 - 4 = 2\ \text{V}$

$R_{\text{eq}} = 2 + 8 = 10\ \Omega$

So, current in the circuit will be

$$\Rightarrow I = \frac{E_{\text{eff}}}{R_{\text{eq}}} \Rightarrow I = \frac{2}{10} = 0.2\ \text{A}$$

Now, potential difference across points X and Y is

$|v_x - v_y| = E + iR$

$$\Rightarrow |v_x - v_y| = 4 + 0.2 \times 8 = 5.6\ \text{V}$$

$$\Rightarrow |v_x - v_y| = 5.6\ \text{v}$$

6. If  $Y, K$  and  $\eta$  are the values of Young's modulus, bulk modulus and modulus of rigidity of any material respectively. Choose the correct relation for these parameters.

$$(1) K = \frac{Y\eta}{9\eta - 3Y} \text{ N/m}^2$$

$$(2) \eta = \frac{3YK}{9K + Y} \text{ N/m}^2$$

$$(3) Y = \frac{9K\eta}{3K - \eta} \text{ N/m}^2$$

$$(4) Y = \frac{9K\eta}{2\eta + 3K} \text{ N/m}^2$$

**Sol. (1)**

$$\Rightarrow y = 3k(1 - 2\sigma)$$

$$\Rightarrow \sigma = \frac{1}{2} \left( 1 - \frac{y}{3k} \right) \quad \dots(1)$$

$$\Rightarrow y = 2\eta(1 + \sigma)$$

$$\Rightarrow \sigma = \frac{y}{2\eta} - 1 \quad \dots(2)$$

by comparing equation (1) and (2), we get

$$\Rightarrow \frac{y}{2\eta} - 1 = \frac{1}{2} \left( 1 - \frac{y}{3k} \right)$$

$$\Rightarrow \frac{y}{\eta} - 2 = 1 - \frac{y}{3k}$$

$$\Rightarrow \frac{y}{\eta} = 1 + 2 - \frac{y}{3k} \Rightarrow \frac{y}{\eta} = 3 - \frac{y}{3k}$$

$$\Rightarrow \frac{y}{3k} = 3 - \frac{y}{\eta} \Rightarrow \frac{y}{3k} = \frac{3\eta - y}{\eta}$$

$$\Rightarrow k = \frac{\eta y}{9\eta - 3y}$$

7. Two stars of masses  $m$  and  $2m$  at a distance  $d$  rotate about their common centre of mass in free space. The period of revolution is -

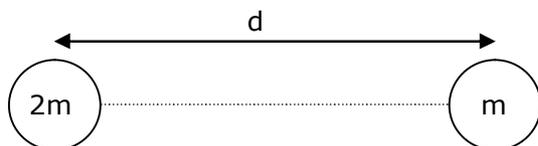
$$(1) 2\pi \sqrt{\frac{d^3}{3Gm}}$$

$$(2) \frac{1}{2\pi} \sqrt{\frac{3Gm}{d^3}}$$

$$(3) \frac{1}{2\pi} \sqrt{\frac{d^3}{3Gm}}$$

$$(4) 2\pi \sqrt{\frac{3Gm}{d^3}}$$

**Sol. (1)**



$$\Rightarrow \frac{G(m)(2m)}{d^2} = m\omega^2 \times \frac{2d}{3}$$

$$\Rightarrow \frac{2Gm}{d^2} = \omega^2 \times \frac{2d}{3}$$

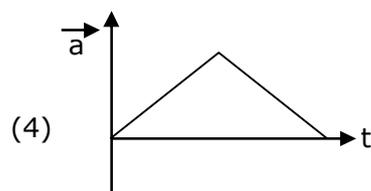
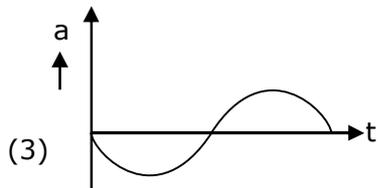
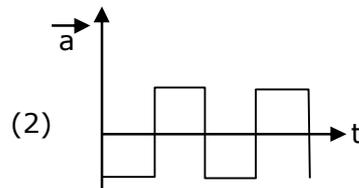
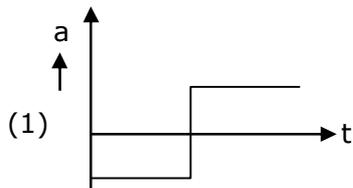
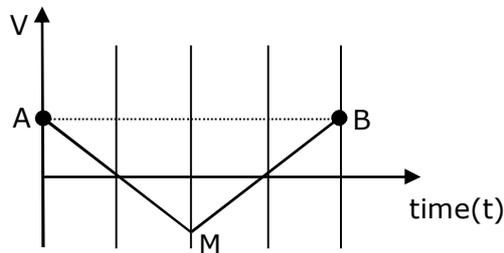
$$\Rightarrow \omega^2 = \frac{3Gm}{d^3}$$

$$\Rightarrow \omega = \sqrt{\frac{3Gm}{d^3}}$$

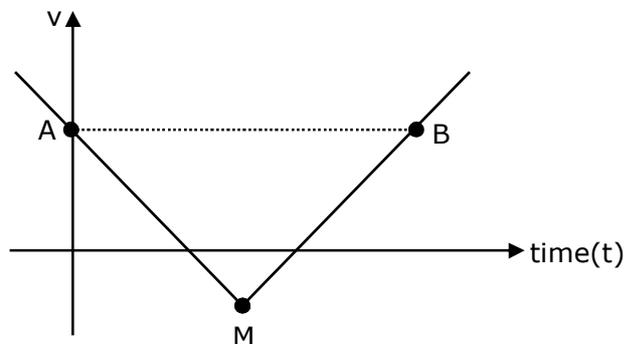
we know that,  $\omega = \frac{2\pi}{T}$  so  $T = \frac{2\pi}{\omega}$

$$\Rightarrow T = \frac{2\pi}{\sqrt{\frac{3Gm}{d^3}}} \Rightarrow T = 2\pi \sqrt{\frac{d^3}{3Gm}}$$

8. If the velocity-time graph has the shape AMB, what would be the shape of the corresponding acceleration-time graph ?



Sol. (1)



$$a = \frac{dv}{dt} = \text{slope of } (v - t) \text{ curve}$$

If  $m = +ve$ , then equation of straight line is  
 $y = mx + c \Rightarrow v = mt + c$  (for MB)

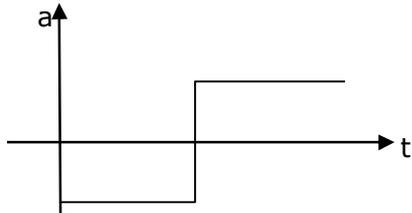
If  $m = -ve$ , then equation of straight line is

$$y = -mx + c \Rightarrow v = -mt + c \text{ (for AM)}$$

If we differentiate equation (1) and (2), we get

$$a_{MB} = +ve = m$$

$a_{AM} = -ve = -m$ , so graph of (a-t) will be



9. Given below are two statements :

Statement - I: Two photons having equal linear momenta have equal wavelengths.

Statement-II: If the wavelength of photon is decreased, then the momentum and energy of a photon will also decrease.

In the light of the above statements, choose the correct answer from the options given below.

- (1) Statement-I is false but Statement-II is true
- (2) Both Statement-I and Statement-II are true
- (3) Both Statement-I and Statement-II are false
- (4) Statement-I is true but Statement-II is false

Sol. (4)

By theory

10. A current through a wire depends on time as  $i = \alpha_0 t + \beta t^2$

where  $\alpha_0 = 20 \text{ A/s}$  and  $\beta = 8 \text{ As}^{-2}$ . Find the charge crossed through a section of the wire in 15 s.

- (1) 2100 C
- (2) 260 C
- (3) 2250 C
- (4) 11250 C

Sol. (4)

$$\text{given : } i = \alpha_0 t + \beta t^2$$

$$\alpha = 20 \text{ A/s and } \beta = 8 \text{ As}^{-2}$$

$$t = 15 \text{ sec}$$

$$\text{we know that, } i = \frac{dq}{dt} \Rightarrow \int_0^t i dt = \int_0^Q dq$$

$$\Rightarrow \int_0^{15} (\alpha_0 t + \beta t^2) dt = \int_0^Q dq$$

$$\Rightarrow Q = \left[ \frac{\alpha_0 t^2}{2} + \frac{\beta t^3}{3} \right]_0^{15}$$

$$\Rightarrow Q = \frac{20 \times 15 \times 15}{2} + \frac{8 \times 15 \times 15 \times 15}{3} - 0$$

$$\Rightarrow Q = 11250 \text{ C}$$

11. match List I with List II

List-I	List-II
(a) Isothermal	(i) Pressure constant
(b) Isochoric	(ii) Temperature constant
(c) Adiabatic	(iii) Volume constant
(d) Isobaric	(iv) Heat content is constant

Choose the correct answer from the options given below -

- (1)(a) - (ii), (b) - (iv), (c) - (iii), (d) - (i)  
 (2)(a) - (ii), (b) - (iii), (c) - (iv), (d) - (i)  
 (3)(a) - (i), (b) - (iii), (c) - (ii), (d) - (iv)  
 (4) (a) - (iii), (b) - (ii), (c) - (i), (d) - (iv)

Sol. (2)

(a)→(ii), (b)→(iii), (c)→(iv), (d)→(i),

By theory

In isothermal process, temperature is constant.

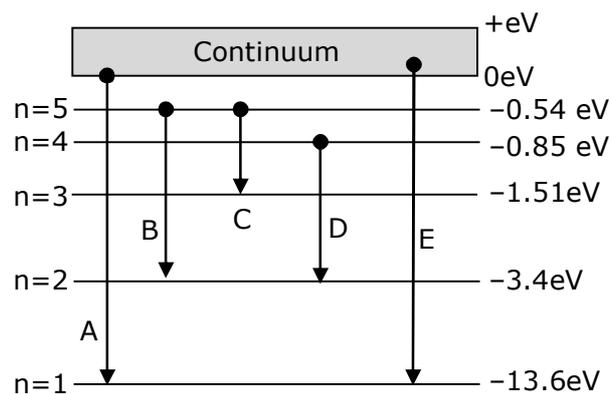
In isochoric process, volume is constant.

In adiabatic process, heat content is constant.

In isobaric process, pressure is constant.

12. In the given figure, the energy levels of hydrogen atom have been shown along with some transitions marked A,B,C,D and E.

The transitions A,B and C respectively represents -



(1)The series limit of Lyman series, third member of balmer series and second member of paschen series

(2)The first member of the Lyman series, third member of Balmer series and second member of paschen series

(3) The ionization potential of hydrogen, second member of Balmer series and third member of Paschen series

(4) The series limit of Lyman series, second member of Balmer series and second member of Paschen series.

**Sol. (1)**

A → series limit of Lyman.

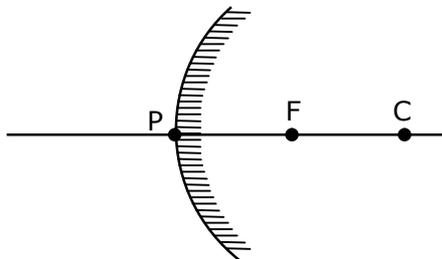
B → 3<sup>rd</sup> member of Balmer series.

C → 2<sup>nd</sup> member of Paschen series.

**13.** The focal length  $f$  is related to the radius of curvature  $r$  of the spherical convex mirror by -

- (1)  $f = r$                       (2)  $f = -\frac{1}{2}r$                       (3)  $f = +\frac{1}{2}r$                       (4)  $f = -r$

**Sol. (3)**



$$\text{So, } \frac{R}{2} = f$$

$$F = +\frac{1}{2} R$$

**14.** Moment of inertia (M.I.) of four bodies, having same mass and radius, are reported as -

$I_1$  = M.I. of thin circular ring about its diameter,

$I_2$  = M.I. of circular disc about an axis perpendicular to disc and going through the centre,

$I_3$  = M.I. of solid cylinder about its axis and

$I_4$  = M.I. of solid sphere about its diameter.

Then :-

(1)  $I_1 = I_2 = I_3 < I_4$                       (2)  $I_1 + I_2 = I_3 + \frac{5}{2} I_4$

(3)  $I_1 + I_3 < I_2 + I_4$                       (4)  $I_1 = I_2 = I_3 > I_4$

**Sol. (4)**

Given ⇒  $I_1$  = M.I. of thin circular ring about its diameter

$I_2$  = M.I. circular disc about an axis perpendicular to disc and going through the centre.

$I_3$  = M.I. of solid cylinder about its axis

$I_4$  = M.I. of solid sphere about its diameter

we know that,

$$I_1 = \frac{MR^2}{2}, I_2 = \frac{MR^2}{2}, I_3 = \frac{MR^2}{2}$$

$$I_4 = \frac{2}{5} MR^2$$

$$\text{So, } I_1 = I_2 = I_3 > I_4$$

- 15.** The workdone by a gas molecule in an isolated system is given by,  $W = \alpha\beta^2 e^{-\frac{x^2}{\alpha kT}}$ , where  $x$  is the displacement,  $k$  is the Boltzmann constant and  $T$  is the temperature.  $\alpha$  and  $\beta$  are constants. Then the dimensions of  $\beta$  will be -

- (1)  $[M^0L^0T^0]$                       (2)  $[M^2LT^2]$                       (3)  $[MLT^{-2}]$                       (4)  $[ML^2T^{-2}]$

**Sol. (3)**

$$\text{given : work} = \alpha.\beta^2.e^{-\frac{x^2}{\alpha.k.T}}$$

$k$  = boltzmann constant

$T$  = temperature

$x$  = displacement

we know that,  $\frac{x^2}{\alpha.k.T}$  = dimensionless

$$\left[ \frac{x^2}{\alpha.k.T} \right] = [M^0L^0T^0]$$

$$[\alpha] = \left[ \frac{L^2}{K.T} \right]$$

$$\Rightarrow [K] = [M^1L^2T^{-2}K^{-1}]$$

$$[T] = [K]$$

$$\Rightarrow [\alpha] = \left[ \frac{L^2}{M^1L^2T^{-2}K^{-1} \times K} \right] \Rightarrow [\alpha] = [M^{-1}T^2]$$

$$\Rightarrow \omega = \alpha.\beta^2$$

$$\Rightarrow \frac{[M^1L^1T^{-2}][L^{-1}]}{[M^{-1}T^2]} = [\beta^2] = [M^2L^2T^{-4}]$$

$$[\beta] = [MLT^{-2}]$$

- 16.** If an emitter current is changed by 4mA, the collector current changes by 3.5 mA. The value of  $\beta$  will be -

- (1) 7                      (2) 0.875                      (3) 0.5                      (4) 3.5

**Sol. (1)**

Given :

$$\Delta I_E = 4 \text{ mA}$$

$$\Delta I_C = 3.5 \text{ mA}$$

we know that,  $\alpha = \frac{\Delta I_C}{\Delta I_E}$

$$\Rightarrow \alpha = \frac{3.5}{4} = \frac{7}{8}$$

Also,  $\beta = \frac{\alpha}{1 - \alpha}$ , so

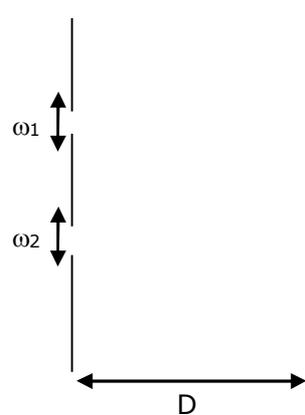
$$\beta = \frac{\frac{7}{8}}{1 - \frac{7}{8}} = \frac{7}{1}$$

$$\beta = 7$$

- 17.** In a Young's double slit experiment, the width of the one of the slit is three times the other slit. The amplitude of the light coming from a slit is proportional to the slit-width. Find the ratio of the maximum to the minimum intensity in the interference pattern.

- (1) 4 : 1                      (2) 2 : 1                      (3) 3 : 1                      (4) 1 : 4

**Sol. (1)**



given :  $\omega_2 = 3\omega_1$

also,  $A \propto \omega$

$$\frac{\omega_1}{\omega_2} = \frac{1}{3} \quad \dots(1)$$

Assume  $\omega_1 = x$ ,  $\omega_2 = 3x$

we know that

$$I_{\max} = (A_1 + A_2)^2, \text{ and}$$

$$I_{\min} = (A_1 - A_2)^2$$

$$\frac{A_1}{A_2} = \frac{\omega_1}{\omega_2} \quad \dots(2)$$

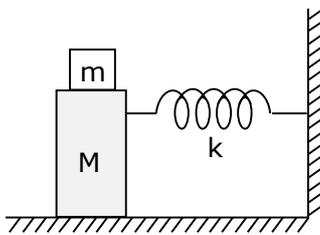
from equation (2) we can say that

$$A_1 = A \text{ and } A_2 = 3A$$

$$\text{Now, } \frac{I_{\max}}{I_{\min}} = \frac{(A + 3A)^2}{(A - 3A)^2} = \frac{16A^2}{4A^2} = \frac{4}{1}$$

$$\Rightarrow \frac{I_{\max}}{I_{\min}} = \frac{4}{1}$$

- 18.** In the given figure, a mass  $M$  is attached to a horizontal spring which is fixed on one side to a rigid support. The spring constant of the spring is  $k$ . The mass oscillates on a frictionless surface with time period  $T$  and amplitude  $A$ . When the mass is in equilibrium position, as shown in the figure, another mass  $m$  is gently fixed upon it. The new amplitude of oscillation will be -



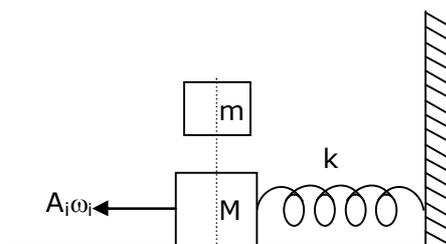
(1)  $A \sqrt{\frac{M}{M+m}}$

(2)  $A \sqrt{\frac{M}{M-m}}$

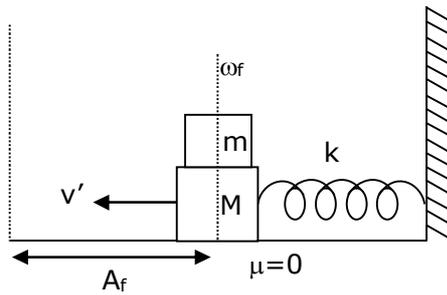
(3)  $A \sqrt{\frac{M-m}{M}}$

(4)  $A \sqrt{\frac{M+m}{M}}$

**Sol. (1)**



Before placing



After placing

We know that  $\omega = \sqrt{\frac{k}{m}}$  and  $\omega_i = \sqrt{\frac{k}{M}}$   $A_i = A$

Also, momentum is conserved just before and just after the block of mass (m) is placed because there is no impulsive force. So -

$$MA_i\omega_i = (M + m) v'$$

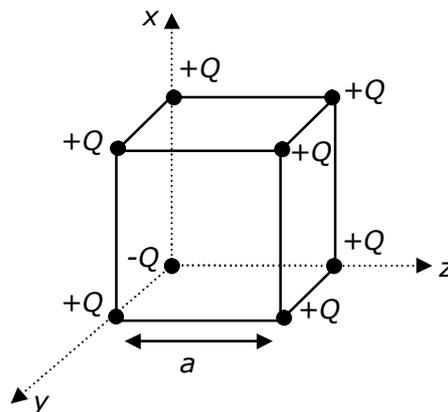
$$v' = \frac{MA_i\omega_i}{(M + m)} \Rightarrow v' = A_f\omega_f$$

$$\frac{MA\omega_i}{(M + m)} = A_f\sqrt{\frac{K}{(M + m)}}$$

$$\Rightarrow \frac{MA\sqrt{\frac{K}{M}}}{M + m} \times \sqrt{\frac{M + m}{K}} = A_f$$

$$\Rightarrow A_f = A\sqrt{\frac{M}{(M + m)}}$$

19. A cube of side 'a' has point charges +Q located at each of its vertices except at the origin where the charge is -Q. The electric field at the centre of cube is :



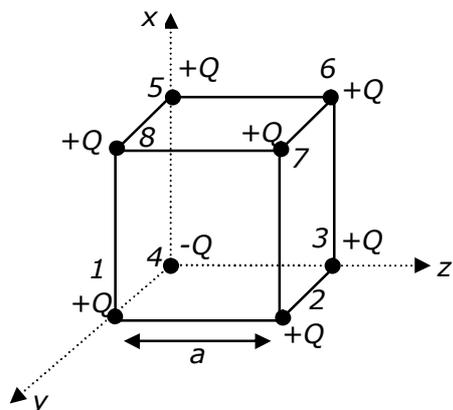
(1)  $\frac{2Q}{3\sqrt{3}\pi\epsilon_0 a^2} (\hat{x} + \hat{y} + \hat{z})$

(2)  $\frac{Q}{3\sqrt{3}\pi\epsilon_0 a^2} (\hat{x} + \hat{y} + \hat{z})$

(3)  $\frac{-2Q}{3\sqrt{3}\pi\epsilon_0 a^2} (\hat{x} + \hat{y} + \hat{z})$

(4)  $\frac{-Q}{3\sqrt{3}\pi\epsilon_0 a^2} (\hat{x} + \hat{y} + \hat{z})$

**Sol. (3)**



If only +Q charges are placed at the corners of cube of side a then electric field at the centre of the cube will be zero.

But in the given condition one (-Q) is placed at one corner of cube so here

$E_1 = E_6, E_2 = E_5$  and  $E_3 = E_8$  (So it will cancel out each other so electric field at centre is due to  $Q_4$  and  $Q_7$ ).

Here electric field at centre = 2 (E.f.)<sub>4</sub>

As,  $|E_4| = |E_7|$

$$(E.F)_C = \frac{2kQ}{\left(\frac{\sqrt{3}a}{2}\right)^2} = \frac{8kQ}{3a^2} \quad \left\{ \because K = \frac{1}{4\pi\epsilon_0} \right\}$$

$$(E.F)_C = \frac{2Q}{3a^2\pi\epsilon_0}$$

$$\text{In vector form } \Rightarrow \vec{E} = \frac{-2Q}{3a^2\pi\epsilon_0} \times \left( \frac{\hat{x} + \hat{y} + \hat{z}}{\sqrt{3}} \right)$$

**20.** Each side of a box made of metal sheet in cubic shape is 'a' at room temperature 'T', the coefficient of linear expansion of the metal sheet is ' $\alpha$ '. The metal sheet is heated uniformly, by a small temperature  $\Delta T$ , so that its new temperature is  $T + \Delta T$ . Calculate the increase in the volume of the metal box-

- (1)  $\frac{4}{3} \pi a^3 \alpha \Delta T$       (2)  $4\pi a^3 \alpha \Delta T$       (3)  $3a^3 \alpha \Delta T$       (4)  $4a^3 \alpha \Delta T$

**Sol. (3)**

volume expansion  $\gamma = 3\alpha$

$$\frac{\Delta V}{V} = \gamma \Delta T$$

$$\Delta V = V \cdot \gamma \Delta T$$

$$\Delta V = a^3 \cdot 3\alpha \Delta T$$

## SECTION-B

1. A resonance circuit having inductance and resistance  $2 \times 10^{-4}$  H and  $6.28 \Omega$  respectively oscillates at 10 MHz frequency. The value of quality factor of this resonator is\_\_\_\_\_.  
 [ $\pi = 3.14$ ]

**Sol. 2000**

Given :  $R = 6.28 \Omega$

$f = 10$  MHz

$L = 2 \times 10^{-4}$  Henry

we know that quality factor Q is given by

$$\Rightarrow Q = \frac{X_L}{R} = \frac{\omega L}{R}$$

also,  $\omega = 2\pi f$ , so

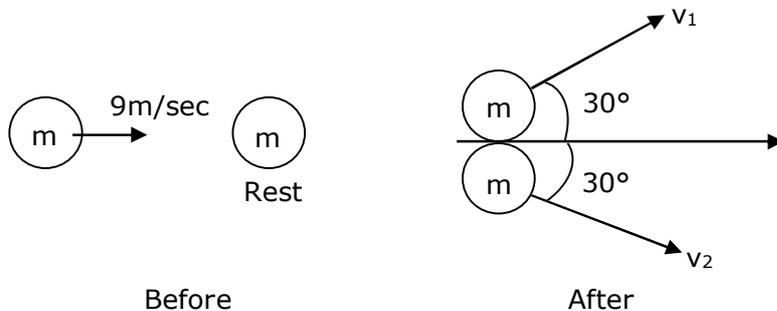
$$\Rightarrow Q = \frac{2\pi f L}{R}$$

$$\Rightarrow Q = \frac{2\pi \times 10 \times 10^6 \times 2 \times 10^{-4}}{6.28} = 2000$$

$Q = 2000$

2. A ball with a speed of 9 m/s collides with another identical ball at rest. After the collision, the direction of each ball makes an angle of  $30^\circ$  with the original direction. The ratio of velocities of the balls after collision is  $x : y$ , where x is \_\_\_\_\_.

**Sol. 1**



Momentum is conserved just before and just after the collision in both x-y direction.

In y-direction

$$p_i = 0$$

$$P_f = m \times \frac{1}{2} v_1 - m \times \frac{1}{2} v_2$$

$p_i = p_f$ , so

$$= \frac{mv_1}{2} - \frac{mv_2}{2} = 0$$

$$\Rightarrow \frac{mv_1}{2} = \frac{mv_2}{2} \Rightarrow v_1 = v_2$$

$$\frac{v_1}{v_2} = 1$$

3. An audio signal  $v_m = 20\sin 2\pi(1500t)$  amplitude modulates a carrier  $v_c = 80 \sin 2\pi (100,000t)$ . The value of percent modulation is \_\_\_\_\_.

**Sol. 25**

$$\text{Given : } v_m = 20 \sin \left[ 100\pi t + \frac{\pi}{4} \right]$$

$$v_c = 80 \sin \left[ 10^4 \pi t + \frac{\pi}{6} \right]$$

$$\text{we know that, modulation index} = \frac{A_m}{A_c}$$

from given equations,  $A_m = 20$  and  $A_c = 80$

$$\text{percentage modulation index} = \frac{A_m}{A_c} \times 100$$

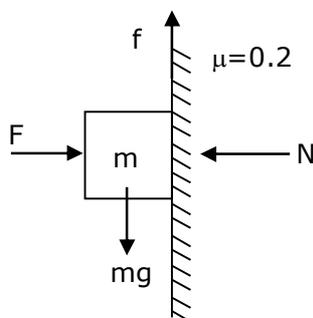
$$\Rightarrow \frac{20}{80} \times 100 = 25\%$$

The value of percentage modulation index is  
=25

4. The coefficient of static friction between a wooden block of mass 0.5 kg and a vertical rough wall is 0.2. The magnitude of horizontal force that should be applied on the block to keep it adhere to the wall will be \_\_\_\_\_ N.

[ $g = 10 \text{ ms}^{-2}$ ]

**Sol. 25**



Given :  $\mu_s = 0.2$

$$m = 0.5 \text{ kg}$$

$$g = 10 \text{ m/s}^2$$

we know that

$$f_s = \mu N \text{ and } \dots(1)$$

To keep the block adhere to the wall

$$\text{here } N = F \dots(2)$$

$$f_s = mg \dots(3)$$

from equation (1), (2), and (3), we get

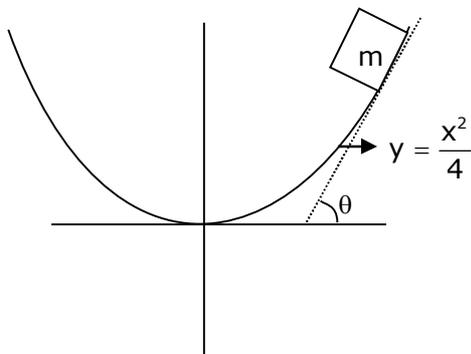
$$\Rightarrow mg = \mu F$$

$$\Rightarrow F = \frac{mg}{\mu} \Rightarrow F = \frac{0.5 \times 10}{0.2}$$

$$F = 25 \text{ N}$$

- 5.** An inclined plane is bent in such a way that the vertical cross-section is given by  $y = \frac{x^2}{4}$  where  $y$  is in vertical and  $x$  in horizontal direction. If the upper surface of this curved plane is rough with coefficient of friction  $\mu = 0.5$ , the maximum height in cm at which a stationary block will not slip downward is \_\_\_\_\_ cm.

**Sol. 25**



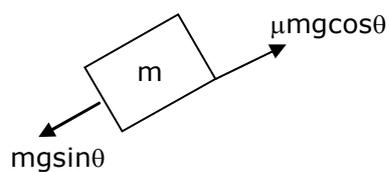
given

$$y = \frac{x^2}{4}$$

$$\mu = 0.5$$

condition for block will not slip downward

$$mg \sin \theta = \mu mg \cos \theta$$



$$\Rightarrow \tan \theta = \mu$$

and we know that

$$\Rightarrow \tan \theta = \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \mu \Rightarrow \frac{x}{2} = 0.5$$

$$\left[ \begin{array}{l} y = \frac{x^2}{4} \\ \frac{dy}{dx} = \frac{x}{2} \end{array} \right]$$

$$\Rightarrow x = 1,$$

put  $x = 1$  in equation  $y = x^2/4$

$$\Rightarrow y = \frac{(1)^2}{4} \Rightarrow y = \frac{1}{4} \Rightarrow y = 0.25$$

$$y = 25 \text{ cm}$$

- 6.** An electromagnetic wave of frequency 5 GHz, is travelling in a medium whose relative electric permittivity and relative magnetic permeability both are 2. Its velocity in this medium is \_\_\_\_\_  $\times 10^7$  m/s.

**Sol. 15**

Given :  $f = 5 \text{ GHz}$

$$\epsilon_r = 2$$

$$\mu_r = 2$$

$$\text{velocity of wave} \Rightarrow v = \frac{c}{n} \quad \dots(1)$$

where,  $n = \sqrt{\mu_r \epsilon_r}$  and  $c = \text{speed of light} = 3 \times 10^8 \text{ m/s}$

$$n = \sqrt{2 \times 2} = 2$$

put the value of  $n$  in we get

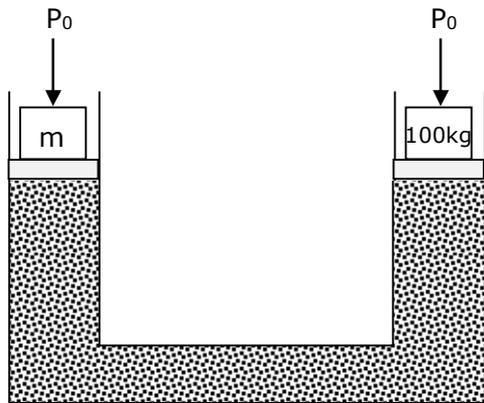
$$\Rightarrow v = \frac{3 \times 10^8}{2} = 15 \times 10^7 \text{ m/s}$$

$$\Rightarrow X \times 10^7 = 15 \times 10^7$$

$$X = 15$$

- 7.** A hydraulic press can lift 100 kg when a mass 'm' is placed on the smaller piston. It can lift \_\_\_\_\_ kg when the diameter of the larger piston is increased by 4 times and that of the smaller piston is decreased by 4 times keeping the same mass 'm' on the smaller piston.

**Sol. 25600**



Atmospheric pressure  $P_0$  will be acting on both the limbs of hydraulic lift.

Applying pascal's law for same liquid level

$$\Rightarrow P_0 + \frac{mg}{A_1} = P_0 + \frac{(100)g}{A_2}$$

$$\Rightarrow \frac{Mg}{A_1} = \frac{(100)g}{A_2} \Rightarrow \frac{m}{100} = \frac{A_1}{A_2} \quad \dots(1)$$

Diameter of piston on side of 100 kg is increased by 4 times so new area =  $16A_2$

Diameter of piston on side of (m) kg is decreasing

$$A_1 = \frac{A_1}{16}$$

(In order to increasing weight lifting capacity, diameter of smaller piston must be reduced)

$$\text{Again, } \frac{mg}{\left(\frac{A_1}{16}\right)} = \frac{M'g}{16A_2} \Rightarrow \frac{256m}{M'} = \frac{A_1}{A_2}$$

$$\text{From equation (1)} = \frac{256m}{M'} = \frac{m}{100} \Rightarrow \therefore M' = 25600 \text{ kg}$$

- 8.** A common transistor radio set requires 12 V (D.C.) for its operation. The D.C. source is constructed by using a transformer and a rectifier circuit, which are operated at 220 V (A.C.) on standard domestic A.C. supply. The number of turns of secondary coil are 24, then the number of turns of primary are \_\_\_\_\_.

**Sol. 440**

Given

Primary voltage,  $V_p = 220 \text{ V}$

Secondary voltage,  $v_s = 12 \text{ V}$

No. of turns in secondary coil is  $N_s = 24$

no. of turns in primary coil,  $N_p = ?$

We know that for a transformer

$$\Rightarrow \frac{N_p}{N_s} = \frac{V_p}{V_s}$$

$$\Rightarrow N_p = \frac{V_p \times N_s}{V_s} = \frac{220 \times 24}{12}$$

$$\Rightarrow N_p = 440$$

9. An unpolarized light beam is incident on the polarizer of a polarization experiment and the intensity of light beam emerging from the analyzer is measured as 100 Lumens. Now, if the analyzer is rotated around the horizontal axis (direction of light) by  $30^\circ$  in clockwise direction, the intensity of emerging light will be \_\_\_\_\_ Lumens.

**Sol. 75**

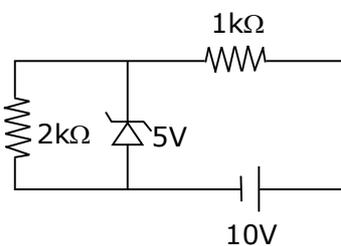
Given :  $I_0 = 100$  lumens,  $\theta = 30^\circ$

$$I_{\text{net}} = I_0 \cos^2 \theta$$

$$I_{\text{net}} = 100 \times \left( \frac{\sqrt{3}}{2} \right)^2 = \frac{100 \times 3}{4}$$

$$I_{\text{net}} = 75 \text{ lumens}$$

10. In connection with the circuit drawn below, the value of current flowing through  $2k\Omega$  resistor is \_\_\_\_\_  $\times 10^{-4}$  A.



**Sol. 25**

In zener diode there will be no change in current after 5V zener diode breakdown

$$\Rightarrow i = \frac{5}{2 \times 10^3}$$

$$\Rightarrow i = 2.5 \times 10^{-3} \text{ A}$$

$$\Rightarrow i = 25 \times 10^{-4} \text{ A}$$

# 24<sup>th</sup> Feb. 2021 | Shift - 1

## CHEMISTRY

### SECTION - A

1. The gas released during anaerobic degradation of vegetation may lead to:

- (1) Global warming and cancer (2) Acid rain  
(3) Corrosion of metals (4) Ozone hole

**Ans. (1)**

**Sol.** Biogas is the mixture of gases produced by the breakdown of organic matter in the absence of oxygen (anaerobically), primarily consisting of methane and carbon dioxide. Biogas can be produced from raw material such as agricultural waste, manure, municipal waste, plant material, sewage, green waste or food waste. Due to release of  $\text{CH}_4$  gas during anaerobic vegetative degradation which causes global warming and cancer.

2. Out of the following, which type of interaction is responsible for the stabilisation  $\alpha$ -helix structure of proteins?

- (1) Ionic bonding (2) Hydrogen bonding  
(3) van der Waals forces (4) Covalent bonding

**Ans. (2)**

**Sol.** The  $\alpha$ -helix is stabilized by hydrogen bond between the NH and CO group of the main chain.

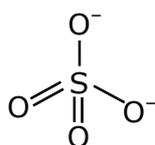
3. Which of the following are isostructural pairs?

- (A)  $\text{SO}_4^{2-}$  and  $\text{CrO}_4^{2-}$   
(B)  $\text{SiCl}_4$  and  $\text{TiCl}_4$   
(C)  $\text{NH}_3$  and  $\text{NO}_3^-$   
(D)  $\text{BCl}_3$  and  $\text{BrCl}_3$

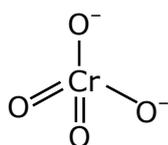
1. A and C only  
2. A and B only  
3. B and C only  
4. C and D only

**Ans. (2)**

**Sol.** (1)  $\text{SO}_4^{2-}$  and  $\text{CrO}_4^{2-}$  both have tetrahedral structure.

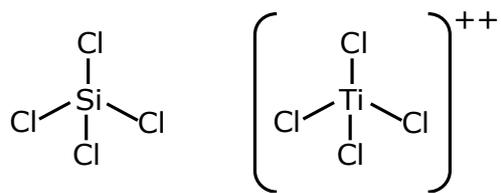


Tetrahedral

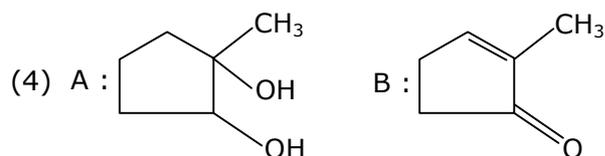
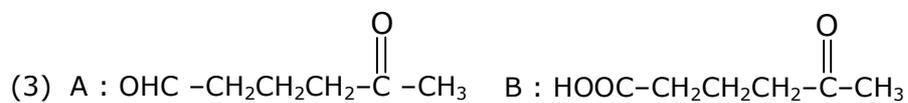
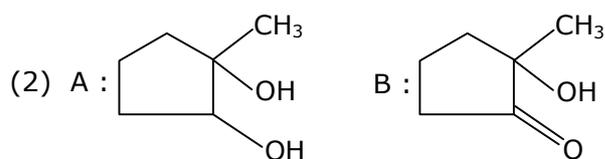
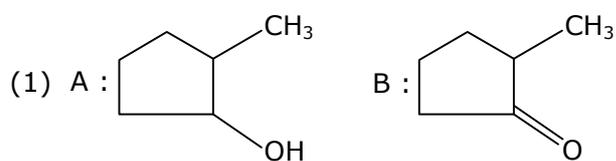
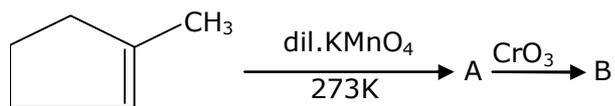


Tetrahedral

(2)  $\text{SiCl}_4$  and  $\text{TiCl}_4$  both have tetrahedral structure also.

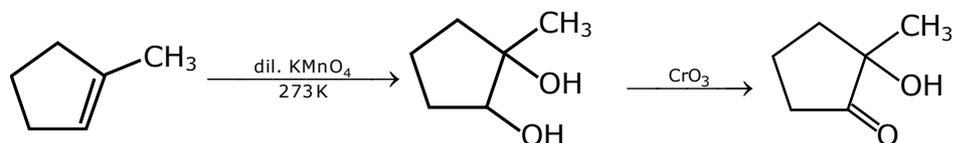


4. Identify products A and B.

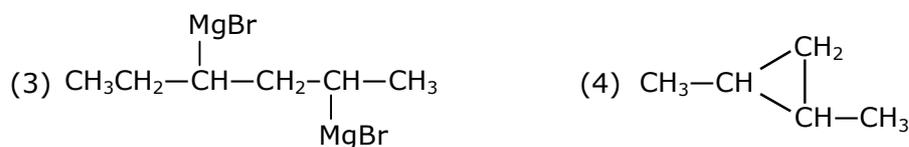
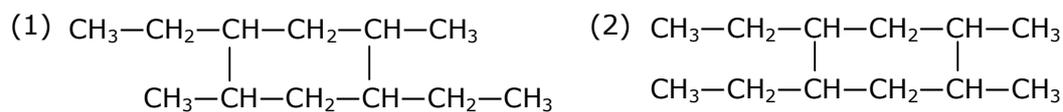
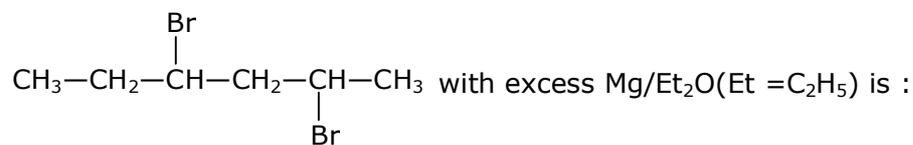


Ans. (2)

Sol.

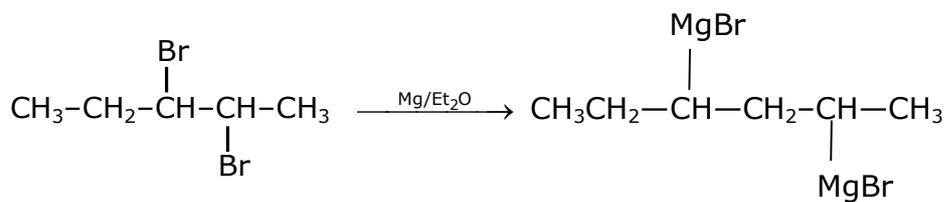


5. The product formed in the first step of the reaction of



Ans. (3)

Sol.



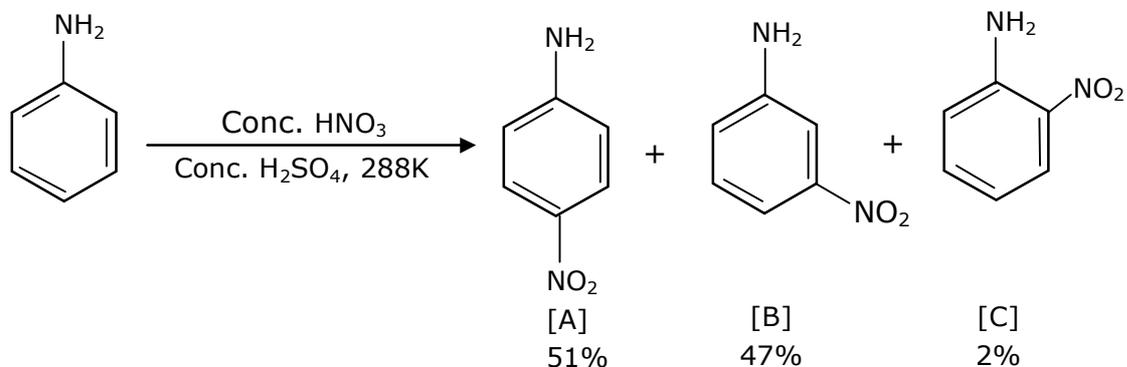
6. The electrode potential of  $\text{M}^{2+}/\text{M}$  of 3d- series elements shows positive value for:

- (1) Zn                      (2) Co                      (3) Fe                      (4) Cu

Ans. (4)

- Sol. (A) Zn                      -0.76  
 (B) CO                      -0.28  
 (C) Fe                      -0.44  
 (D) Cu                      +0.34

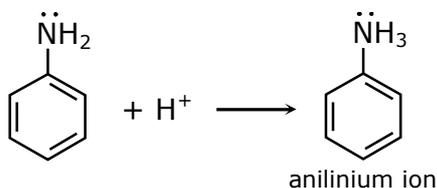
7. In the following reaction the reason why meta-nitro product also formed is:



- (1) Formation of anilinium ion
- (2)  $-\text{NO}_2$  substitution always takes place at meta-position
- (3) low temperature
- (4)  $-\text{NH}_2$  group is highly meta-directive

Ans. (1)

Sol.



In acidic medium the  $-\text{NH}_2$  group in aniline converts into anilinium ion which is meta directing.

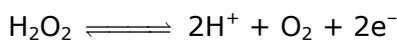
8. (A)  $\text{HOCl} + \text{H}_2\text{O}_2 \rightarrow \text{H}_3\text{O}^+ + \text{Cl}^- + \text{O}_2$   
 (B)  $\text{I}_2 + \text{H}_2\text{O}_2 + 2\text{OH}^- \rightarrow 2\text{I}^- + 2\text{H}_2\text{O} + \text{O}_2$

Choose the correct option.

- (1)  $\text{H}_2\text{O}_2$  act as oxidizing and reducing agent respectively in equations (A) and (B).
- (2)  $\text{H}_2\text{O}_2$  acts as oxidizing agent in equations (A) and (B).
- (3)  $\text{H}_2\text{O}_2$  acts as reducing agent in equations (A) and (B).
- (4)  $\text{H}_2\text{O}_2$  acts as reducing and oxidising agent respectively in equation (A) and (B).

Ans. (3)

Sol. When  $\text{H}_2\text{O}_2$  acts a reducing agent it liberates the  $\text{O}_2$ .

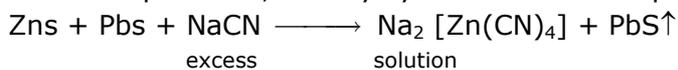


9. Which of the following ore is concentrated using group 1 cyanide salt ?

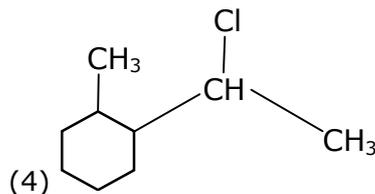
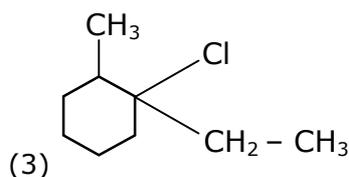
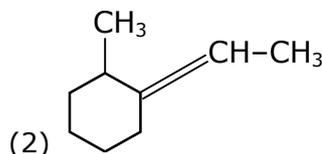
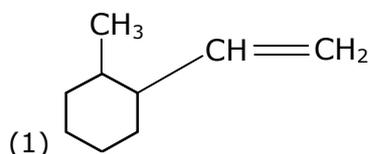
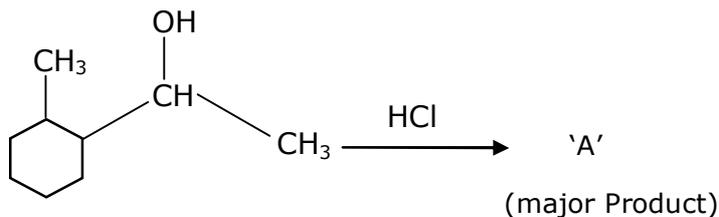
- (1) Sphalerite (2) Siderite  
(3) Malachite (4) Calamine

Ans. (1)

Sol. Conc. of sphalerite, first by cyanide salt as a depressant to remove the impurity of galena

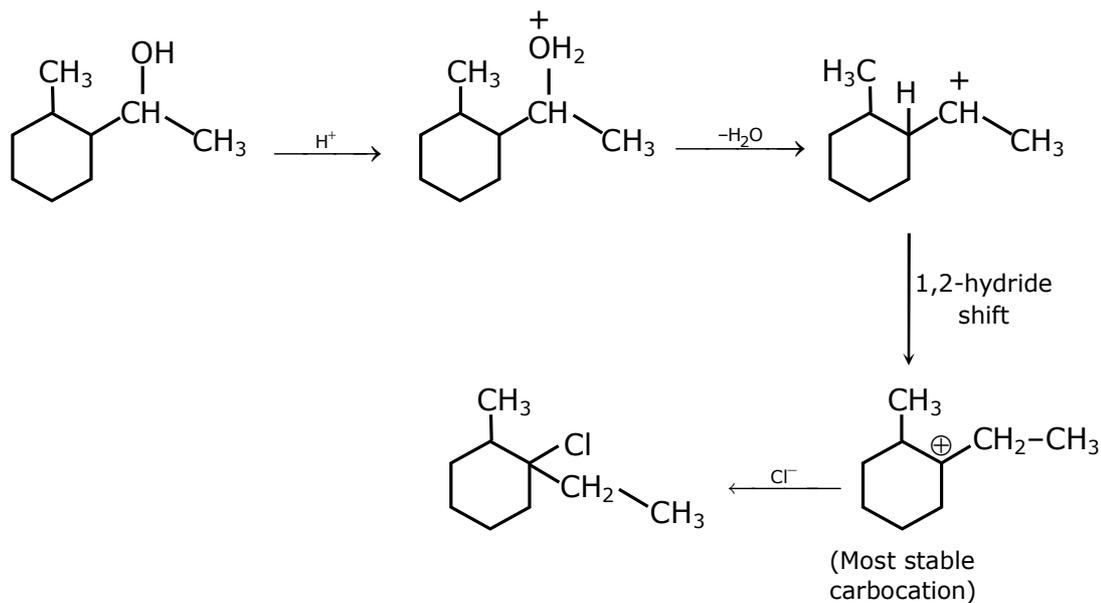


10. Which is the final product (major) 'A' in the given reaction?

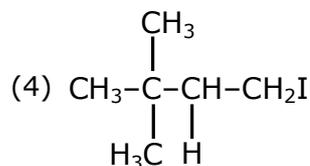
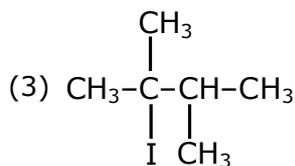
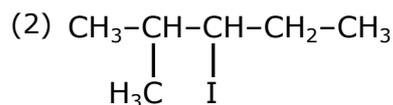
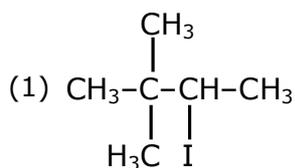


Ans. (3)

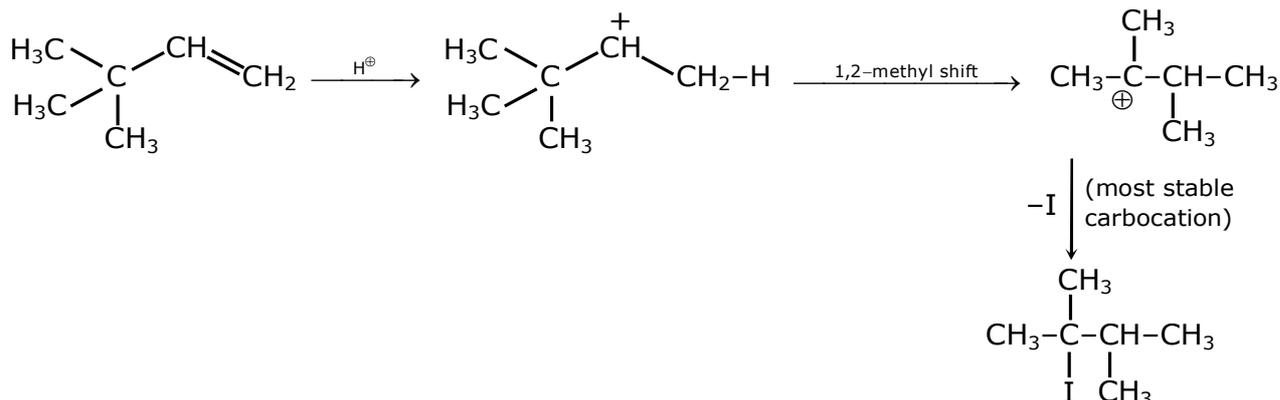
Sol.



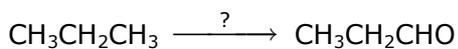
11. What is the major product formed by HI on reaction with  $\text{CH}_3-\overset{\text{CH}_3}{\underset{\text{H}_3\text{C}}{\text{C}}}-\text{CH}=\text{CH}_2$  ?



Ans. (3)  
Sol.



12. Which of the following reagent is used for the following reaction ?

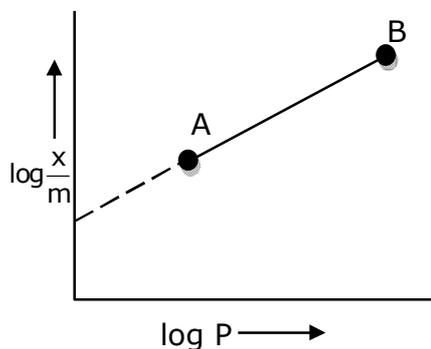


- (1) Potassium permanganate
- (2) Molybdenum oxide
- (3) Copper at high temperature and pressure
- (4) Manganese acetate

Ans. (2)



13. In Freundlich adsorption isotherm, slope of AB line is :



(1)  $\frac{1}{n}$  with  $\left(\frac{1}{n} = 0 \text{ to } 1\right)$

(2)  $\log \frac{1}{n}$  with  $(n < 1)$

(3)  $\log n$  with  $(n > 1)$

(4)  $n$  with  $(n, 0.1 \text{ to } 0.5)$

Ans. (1)

Sol. Freundlich adsorption isotherm is :

$$\frac{x}{m} = kp^{1/n}$$

$x$  = mass of adsorbate

$m$  = mass of adsorbent

$P$  = eq. pressure

$$k_1 n = \frac{1}{n} \log p + \log k$$

$$y = mx + c$$

comparing

$$m = \frac{1}{n} = \text{slope} \left[\frac{1}{n} = 0 \text{ to } 1\right]$$

$$n > 1$$

14. The major components in "Gun Metal" are:

(1) Al, Cu, Mg and Mn

(2) Cu, Sn and Zn

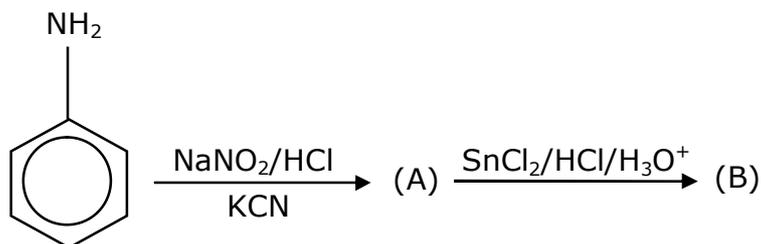
(3) Cu, Zn and Ni

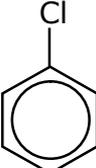
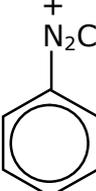
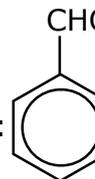
(4) Cu, Ni and Fe

Ans. (2)

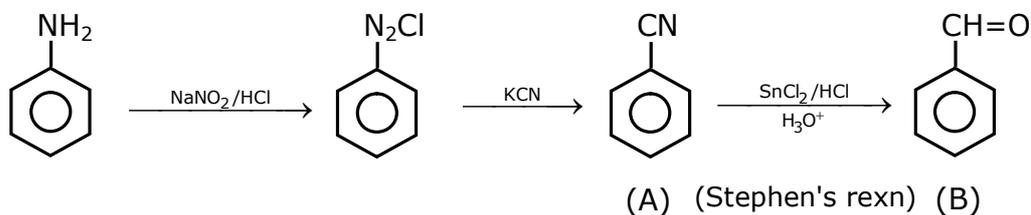
Sol. "Gun metal" is alloy of copper with tin and zinc.

15. 'A' and 'B' in the following reactions are :

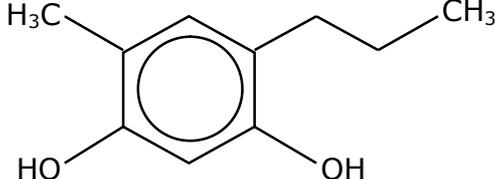
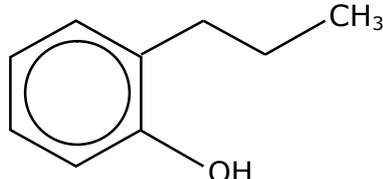
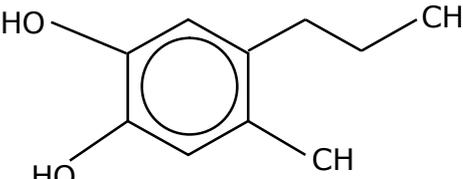
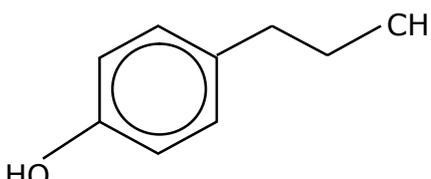


- (1) (A) :  (B) :  (2) (A) :  (B) : 
- (3) (A) :  (B) :  (4) (A) :  (B) : 

Ans. (3)  
Sol.

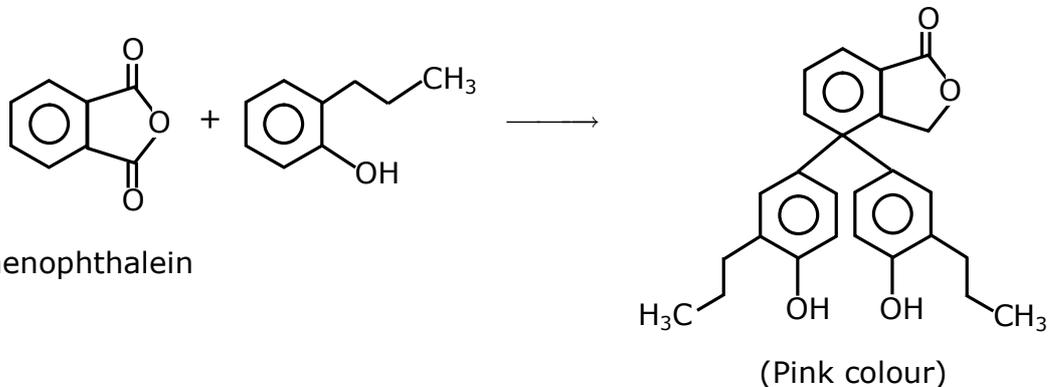


16. Which of the following compound gives pink colour on reaction with phthalic anhydride in conc.  $\text{H}_2\text{SO}_4$  followed by treatment with  $\text{NaOH}$  ?

- (1)  (2) 
- (3)  (4) 

**Ans. (2)**

**Sol.**



**17.** Consider the elements Mg, Al, S, P and Si, the correct increasing order of their first ionization enthalpy is:

(1) Al < Mg < Si < S < P

(2) Al < Mg < S < Si < P

(3) Mg < Al < Si < S < P

(4) Mg < Al < Si < P < S

**Ans. (1)**

**Sol.** Order of IE, in 3<sup>rd</sup> period is

Na < Mg > Al < Si < P > S < Cl < Ar

Na < Al < Mg < Si < S < P < Cl < Ar

due to stable full filed 3s-orbital and more penetrating power

due to half filled 3p<sup>3</sup>-orbital of phosphorous

**18.** Given below are two statements :

Statement I : Colourless cupric metaborate is reduced to cuprous metaborate in a luminous flame.

Statement II : Cuprous metaborate is obtained by heating boric anhydride and copper sulphate in a non-luminous flame.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Statement I is false but statement II is true.

(2) Statement I is true but Statement II is false.

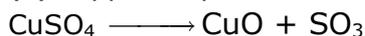
(3) Both Statement I and Statement II are true.

(4) Both Statement I and Statement II are false.

**Ans. (4)**

**Sol.** Both are False

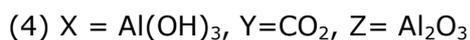
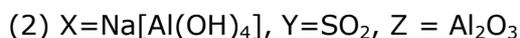
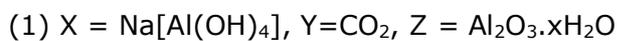
(1) Copper sulphate form copper meta boric with boric anhydride



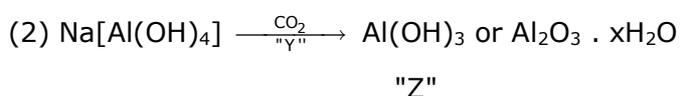
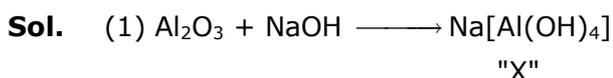
blue in cold oxidising flame (non luminous flame)

(2) Blue coloured metal borate is reduced to copper in a luminous flame.

**19.**  $\text{Al}_2\text{O}_3$  was leached with alkali to get X. The solution of X on passing of gas Y, forms Z. X, Y and Z respectively are :



**Ans. (1)**



**20.** Match List I with List II.

**List I**

(Monomer Unit)

(a) Caprolactum

(b) 2-Chloro-1,3-butadiene

(c) Isoprene

(d) Acrylonitrile

**List II**

(Polymer)

(i) Natural rubber

(ii) Buna-N

(iii) Nylon 6

(iv) Neoprene

Choose the correct answer from the options given below :

(1) (a)  $\rightarrow$  (iii), (b)  $\rightarrow$  (iv), (c)  $\rightarrow$  (i), (d)  $\rightarrow$  (ii)

(2) (a)  $\rightarrow$  (i), (b)  $\rightarrow$  (ii), (c)  $\rightarrow$  (iii), (d)  $\rightarrow$  (iv)

(3) (a)  $\rightarrow$  (ii), (b)  $\rightarrow$  (i), (c)  $\rightarrow$  (iv), (d)  $\rightarrow$  (iii)

(4) (a)  $\rightarrow$  (iv), (b)  $\rightarrow$  (iii), (c)  $\rightarrow$  (ii), (d)  $\rightarrow$  (i)

**Ans. (1)**

**Sol.** (1) Polymer of caprolactum is nylon-6

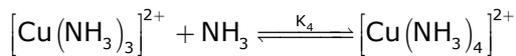
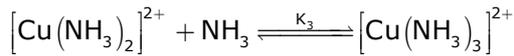
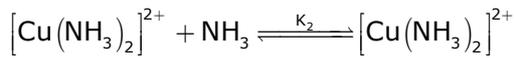
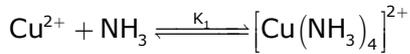
(2) Polymer of 2-chloro-1,3-butadiene is neoprene.

(3) Polymer of isoprene is natural rubber

(4) Polymer of acrylonitrile and 1,3-butadiene is buna-N

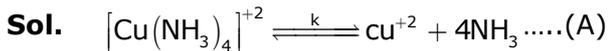
## SECTION – B

1. The stepwise formation of  $[\text{Cu}(\text{NH}_3)]^{2+}$  is given below:

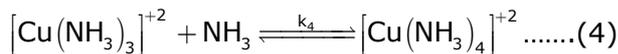
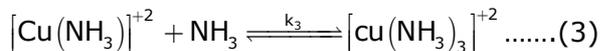
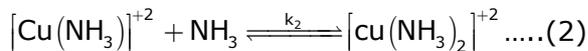
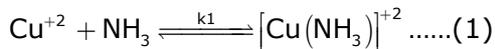


The value of stability constants  $K_1$ ,  $K_2$ ,  $K_3$  and  $K_4$  are  $10^4$ ,  $1.58 \times 10^2$ ,  $5 \times 10^2$  and  $10^2$  respectively. The overall equilibrium constants for dissociation of  $[\text{Cu}(\text{NH}_3)_4]^{2+}$  is  $x \times 10^{-12}$ . The value of  $x$  is \_\_\_\_\_. (Rounded off to the nearest integer)

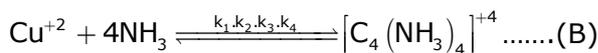
**Ans. (1)**



For this :



(1) + (2) + (3) + (4)



So for (A)

$$K = \frac{1}{k_1 \cdot k_2 \cdot k_3 \cdot k_4}$$

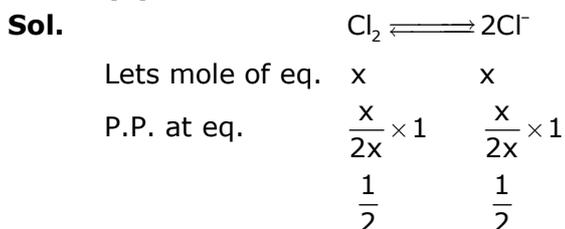
Putting the value of  $k_1, k_2, k_3$  and  $k_4$ .

$$K = \frac{1}{(10^4) \cdot (1.58 \times 10^2) (5 \times 10^2) (10^2)} = 1.26 \times 10^{-12}$$

$$x = 1.$$

2. At 1990 K and 1 atm pressure, there are equal number of  $\text{Cl}_2$  molecules and Cl atoms in the reaction mixture. The value of  $K_p$  for the reaction  $\text{Cl}_{2(g)} \rightleftharpoons 2\text{Cl}_{(g)}$  under the above conditions is  $x \times 10^{-1}$ . The value of  $x$  is \_\_\_\_\_. (Rounded off to the nearest integer)

**Ans. (5)**



$$K_p = \frac{[\text{P}_{\text{Cl}}]^2}{[\text{P}_{\text{Cl}_2}]} = \frac{\left[\frac{1}{2}\right]^2}{\frac{1}{2}} = \frac{1}{2} = 0.5 = 5 \times 10^{-1}$$

$$X = 5.$$

3. 4.5 g of compound A (MW = 90) was used to make 250 mL of its aqueous solution. The molarity of the solution in M is  $x \times 10^{-1}$ . The value of  $x$  is \_\_\_\_\_. (Rounded off to the nearest integer)

**Ans. (2)**

**Sol.** Moles of A =  $\frac{\text{Weight}}{\text{M.w}}$

$$= \frac{4.5}{90} = \frac{1}{20} = 0.05$$

Volume (Lit) =  $\frac{250}{1000} = 0.250$  lit lit

Moles of A =  $\frac{\text{Weight}}{\text{M.w}}$

$$= \frac{4.5}{90} = \frac{1}{20} = 0.05$$

Volume (Lit) =  $\frac{250}{1000} = 0.250$  lit lit

Molarity (M) =  $\frac{\text{Mole}}{(\text{Lit})\text{volume}} = \frac{0.05}{0.250} = 0.2$

$$= 2 \times 10^{-1} \frac{\text{mol}}{\text{Lit}}$$

$x = 2$

Molarity (M) =  $\frac{\text{Mole}}{(\text{Lit})\text{volume}} = \frac{0.05}{0.250} = 0.2$

$$= 2 \times 10^{-1} \frac{\text{mol}}{\text{Lit}} \quad x = 2$$



7. A proton and a  $\text{Li}^{3+}$  nucleus are accelerated by the same potential. If  $\lambda_{\text{Li}}$  and  $\lambda_p$  denote the de Broglie wavelengths of  $\text{Li}^{3+}$  and proton respectively, then the value of  $\frac{\lambda_{\text{Li}}}{\lambda_p}$  is  $x \times 10^{-1}$ . The value of  $x$  is \_\_\_\_\_. [Rounded off to the nearest integer]  
[Mass of  $\text{Li}^{3+} = 8.3$  mass of proton]

**Ans. (2)**

**Sol.** De Broglie Wavelength

$$\lambda = \frac{h}{\sqrt{2m \text{ k.E.}}}$$

$$\frac{\lambda_{\text{Li}^{3+}}}{\lambda_p} = \frac{\sqrt{m_p \times (e^- v)_p}}{\sqrt{m_{\text{Li}^{3+}} \times 3e_p v}}$$

$$m_{\text{Li}^{3+}} = 8.3 m_p$$

$$\frac{\lambda_{\text{Li}^{3+}}}{\lambda_p} = \sqrt{\frac{m_p}{3 \times 8.3 m_p}} = \sqrt{\frac{1}{25}}$$

$$= \frac{1}{5} = 0.2 = 2 \times 10^{-1}$$

$$x = 2.$$

8. Gaseous cyclobutene isomerizes to butadiene in a first order process which has a 'k' value of  $3.3 \times 10^{-4} \text{ s}^{-1}$  at  $153^\circ\text{C}$ . The time in minutes it takes for the isomerization to proceed 40% to completion at this temperature is \_\_\_\_\_. (Rounded off to the nearest integer)

**Ans. (26)**

**Sol.** For first order Rxn :-

$$t = \frac{2.303}{k} \log \left[ \frac{100}{100 - x} \right]$$

$$X = 40, k = 3.3 \times 10^{-4}$$

$$t = \frac{2.303}{3.3 \times 10^{-4}} \log \left[ \frac{100}{60} \right]$$

For first order Rxn :-

$$t = \frac{2.303}{k} \log \left[ \frac{100}{100 - x} \right]$$

$$X = 40, k = 3.3 \times 10^{-4}$$

$$t = \frac{2.303}{3.3 \times 10^{-4}} \log \left[ \frac{100}{60} \right]$$

$$t = \frac{2.303}{3.3 \times 10^{-4}} \times 0.22$$

$$t = 0.1535.3 \times 10^4$$

$$t = 1535 \text{ sec.}$$

$$t = 0.1535.3 \times 10^4$$

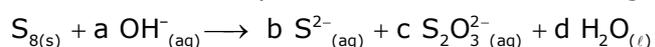
$$t = 1535 \text{ sec} = 25.6 \text{ Min.}$$

9. For the reaction  $A_{(g)} \rightarrow B_{(g)}$ , the value of the equilibrium constant at 300 K and 1 atm is equal to 100.0. The value of  $\Delta_r G$  for the reaction at 300 K and 1 atm in  $\text{J mol}^{-1}$  is  $-xR$ , where  $x$  is \_\_\_\_\_.  
(Rounded off to the nearest integer)  
[ $R = 8.31 \text{ J mol}^{-1}\text{K}^{-1}$  and  $\ln 10 = 2.3$ ]

**Ans. (1380)**

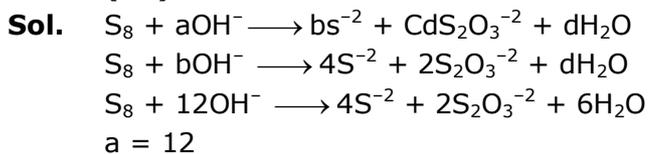
**Sol.**  $\Delta G^\circ = -RT \ln K_{eq}$   
 $= -R \times 300 \times \ln(10^2)$   
 $= 300 \times 2 \times 2.3 \times (-R)$   
 $= -1380R$   
 $x = 1380$  ans.

10. The reaction of sulphur in alkaline medium is given below:



The values of 'a' is \_\_\_\_\_. (Integer answer)

**Ans. (12)**



# 24<sup>th</sup> Feb. 2021 | Shift - 1

## MATHEMATICS

1. The locus of the mid-point of the line segment joining the focus of the parabola  $y^2=4ax$  to a moving point of the parabola, is another parabola whose directrix is:

- (1)  $x = a$                       (2)  $x = 0$                       (3)  $x = -\frac{a}{2}$                       (4)  $x = \frac{a}{2}$

**Ans. (2)**

Sol.  $h = \frac{at^2 + a}{2}, k = \frac{2at + 0}{2}$

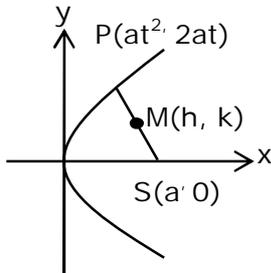
$$\Rightarrow t^2 = \frac{2h - a}{a} \text{ and } t = \frac{k}{a}$$

$$\Rightarrow \frac{k^2}{a^2} = \frac{2h - a}{a}$$

$$\Rightarrow \text{Locus of } (h, k) \text{ is } y^2 = a(2x - a)$$

$$\Rightarrow y^2 = 2a\left(x - \frac{a}{2}\right)$$

$$\text{Its directrix is } x - \frac{a}{2} = -\frac{a}{2} \Rightarrow x = 0$$



2. A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed is:

- (1) 560                      (2) 1050                      (3) 1625                      (4) 575

**Ans. (3)**

Sol.  $(2I, 4F) + (3I, 6F) + (4I, 8F)$   
 $= {}^6C_2 {}^8C_4 + {}^6C_3 {}^8C_6 + {}^6C_4 {}^8C_8$   
 $= 15 \times 70 + 20 \times 28 + 15 \times 1$   
 $= 1050 + 560 + 15 = 1625$

3. The equation of the plane passing through the point (1, 2, -3) and perpendicular to the planes  $3x + y - 2z = 5$  and  $2x - 5y - z = 7$ , is:

(1)  $3x - 10y - 2z + 11 = 0$

(2)  $6x - 5y - 2z - 2 = 0$

(3)  $11x + y + 17z + 38 = 0$

(4)  $6x - 5y + 2z + 10 = 0$

Ans. (3)

Sol. Normal vector of required plane is  $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix} = -11\hat{i} - \hat{j} - 17\hat{k}$

$\therefore 11(x - 1) + (y - 2) + 17(z + 3) = 0$

$11x + y + 17z + 38 = 0$

4. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is  $\frac{1}{4}$ . Three stones A, B and C are placed at the points (1, 1), (2, 2) and (4, 4) respectively. Then which of these stones is/are on the path of the man?

(1) B only (2) A only (3) All the three (4) C only

Ans. (1)

Sol.  $\frac{x}{a} + \frac{y}{b} = 1$

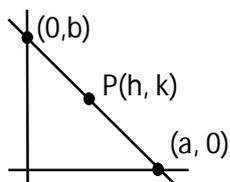
$\frac{h}{a} + \frac{k}{b} = 1$  .....(1)

$\frac{\frac{1}{a} + \frac{1}{b}}{2} = \frac{1}{4}$

$\therefore \frac{1}{a} + \frac{1}{b} = \frac{1}{2}$  .....(ii)

$\therefore$  Line passes through fixed point B(2, 2)

(from (1) and (2))



5. The statement among the following that is a tautology is:

- (1)  $A \wedge (A \vee B)$       (2)  $B \rightarrow [A \wedge (A \rightarrow B)]$       (3)  $A \vee (A \wedge B)$       (4)  $[A \wedge (A \rightarrow B)] \rightarrow B$

**Ans. (4)**

Sol.  $A \wedge (\sim A \vee B) \rightarrow B$   
 $= [(A \wedge \sim A) \vee (A \wedge B)] \rightarrow B$   
 $= (A \wedge B) \rightarrow B$   
 $= \sim A \vee \sim B \vee B$   
 $= t$

6. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined as  $f(x) = 2x - 1$  and  $g: \mathbb{R} - \{1\} \rightarrow \mathbb{R}$  be defined as  $g(x) = \frac{x - \frac{1}{2}}{x - 1}$ .

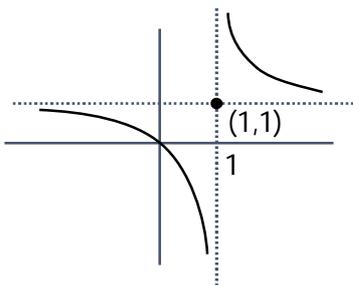
Then the composition function  $f(g(x))$  is :

- (1) both one-one and onto      (2) onto but not one-one  
 (3) neither one-one nor onto      (4) one-one but not onto

**Ans. (4)**

Sol.  $f(g(x)) = 2g(x) - 1$   
 $= 2 \left( \frac{x - \frac{1}{2}}{x - 1} \right) - 1 = \frac{x}{x - 1}$   
 $f(g(x)) = 1 + \frac{1}{x - 1}$

one-one, into





**Ans. (1)**

Sol.  $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2} = \lambda$

$\Rightarrow x = \lambda+3, y = 2\lambda+4, z = 2\lambda+5$

Which lines on given plane hence

$\Rightarrow \lambda + 3 + 2\lambda + 4 + 2\lambda + 5 = 17$

$\Rightarrow \lambda = \frac{5}{5} = 1$

Hence, point of intersection is Q (4, 6, 7)

$\therefore$  Required distance = PQ

$= \sqrt{9+25+4}$

$= \sqrt{38}$

10.  $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} (\sin \sqrt{t}) dt}{x^3}$  is equal to :

(1)  $\frac{2}{3}$

(2) 0

(3)  $\frac{1}{15}$

(4)  $\frac{3}{2}$

**Ans. (1)**

Sol.  $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \sin \sqrt{t} dt}{x^3} = \lim_{x \rightarrow 0} \frac{(\sin |x|) 2x}{3x^2} = \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right) \times \frac{2}{3} = \frac{2}{3}$

11. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is:

(1) 25

(2)  $20\sqrt{3}$

(3) 30

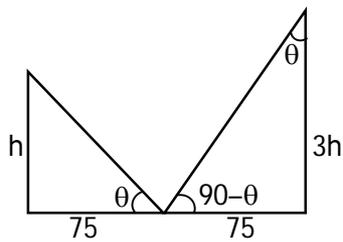
(4)  $25\sqrt{3}$

**Ans. (4)**

Sol.  $\tan \theta = \frac{h}{75} = \frac{75}{3h}$

$\Rightarrow h^2 = \frac{(75)^2}{3}$

$h = 25\sqrt{3}m$



12. If the tangent to the curve  $y = x^3$  at the point  $P(t, t^3)$  meets the curve again at  $Q$ , then the ordinate of the point which divides  $PQ$  internally in the ratio  $1 : 2$  is :

- (1)  $-2t^3$                       (2)  $-t^3$                       (3)  $0$                       (4)  $2t^3$

Ans. (1)

Sol. Equation of tangent at  $P(t, t^3)$

$$(y - t^3) = 3t^2(x - t) \quad \dots\dots(1)$$

Now solve the above equation with

$$y = x^3 \quad \dots\dots(2)$$

By (1) & (2)

$$x^3 - t^3 = 3t^2(x - t)$$

$$x^2 + xt + t^2 = 3t^2$$

$$x^2 + xt - 2t^2 = 0$$

$$(x - t)(x + 2t) = 0$$

$$\Rightarrow x = -2t \Rightarrow Q(-2t, -8t^3)$$

$$\text{Ordinate of required point} = \frac{2t^3 + (-8t^3)}{3} = -2t^3$$

13. The area (in sq. units) of the part of the circle  $x^2+y^2=36$ , which is outside the parabola  $y^2=9x$ , is :

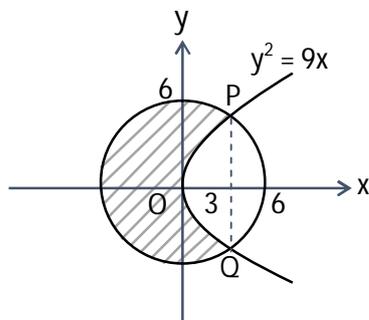
- (1)  $24\pi + 3\sqrt{3}$   
 (2)  $12\pi + 3\sqrt{3}$   
 (3)  $12\pi - 3\sqrt{3}$   
 (4)  $24\pi - 3\sqrt{3}$

**Ans. (4)**

Sol. The curves intersect at point  $(3, \pm 3\sqrt{3})$

Required area

$$\begin{aligned}
 &= \pi r^2 - 2 \left[ \int_0^3 \sqrt{9x} dx + \int_3^6 \sqrt{36-x^2} dx \right] \\
 &= 36\pi - 12\sqrt{3} - 2 \left( \frac{x}{2} \sqrt{36-x^2} + 18 \sin^{-1} \left( \frac{x}{6} \right) \right) \Bigg|_3^6 \\
 &= 36\pi - 12\sqrt{3} - 2 \left( 9 - \left( \frac{9\sqrt{3}}{2} + 3\pi \right) \right) = 24\pi - 3\sqrt{3}
 \end{aligned}$$



14. If  $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a \sin^{-1} \left( \frac{\sin x + \cos x}{b} \right) + c$ , where  $c$  is a constant of integration, then the ordered pair  $(a, b)$  is equal to :

- (1)  $(1, -3)$                       (2)  $(1, 3)$                       (3)  $(-1, 3)$                       (4)  $(3, 1)$

**Ans. (2)**

Sol. put  $\sin x + \cos x = t \Rightarrow 1 + \sin 2x = t^2$

$$\Rightarrow (\cos x - \sin x) dx = dt$$

$$\therefore I = \int \frac{dt}{\sqrt{8 - (t^2 - 1)}} = \int \frac{dt}{\sqrt{9 - t^2}} = \sin^{-1} \left( \frac{t}{3} \right) + C = \sin^{-1} \left( \frac{\sin x + \cos x}{3} \right) + C$$

$$\Rightarrow a = 1 \text{ and } b = 3$$

15. The population  $P = P(t)$  at time 't' of a certain species follows the differential equation  $\frac{dP}{dt} = 0.5P - 450$ . If  $P(0) = 850$ , then the time at which population becomes zero is :

- (1)  $\frac{1}{2} \log_e 18$                       (2)  $2 \log_e 18$                       (3)  $\log_e 9$                       (4)  $\log_e 18$

Ans. (2)

Sol.  $\frac{dp}{dt} = \frac{p - 900}{2}$

$$\int_{850}^0 \frac{dp}{p - 900} = \int_0^t \frac{dt}{2}$$

$$\ln|p - 900| \Big|_{850}^0 = \frac{t}{2}$$

$$\ln|900| - \ln|50| = \frac{t}{2}$$

$$\frac{t}{2} = \ln|18|$$

$$\Rightarrow t = 2 \ln 18$$

16. The value of  $-^{15}C_1 + 2 \cdot ^{15}C_2 - 3 \cdot ^{15}C_3 + \dots - 15 \cdot ^{15}C_{15} + ^{14}C_1 + ^{14}C_3 + ^{14}C_5 + \dots + ^{14}C_{11}$  is:

- (1)  $2^{14}$                       (2)  $2^{13} - 13$                       (3)  $2^{16} - 1$                       (4)  $2^{13} - 14$

Ans. (4)

Sol.  $S_1 = -^{15}C_1 + 2 \cdot ^{15}C_2 - \dots - 15 \cdot ^{15}C_{15}$

$$= \sum_{r=1}^{15} (-1)^r \cdot r \cdot ^{15}C_r = 15 \sum_{r=1}^{15} (-1)^r \cdot ^{14}C_{r-1}$$

$$= 15 (-^{14}C_0 + ^{14}C_1 - \dots - ^{14}C_{14}) = 15 (0) = 0$$

$$S_2 = ^{14}C_1 + ^{14}C_3 + \dots + ^{14}C_{11}$$

$$= (^{14}C_1 + ^{14}C_3 + \dots + ^{14}C_{11} + ^{14}C_{13}) - ^{14}C_{13}$$

$$= 2^{13} - 14$$

$$= S_1 + S_2 = 2^{13} - 14$$

17. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is :

- (1)  $\frac{3}{16}$                       (2)  $\frac{1}{2}$                       (3)  $\frac{5}{16}$                       (4)  $\frac{1}{32}$

**Ans. (2)**

Sol.  $P(\text{odd no. twice}) = P(\text{even no. thrice})$

$$\Rightarrow {}^n C_2 \left(\frac{1}{2}\right)^n = {}^n C_3 \left(\frac{1}{2}\right)^n \Rightarrow n = 5$$

Success is getting an odd number then  $P(\text{odd successes}) = P(1) + P(3) + P(5)$

$$= {}^5 C_1 \left(\frac{1}{2}\right)^5 + {}^5 C_3 \left(\frac{1}{2}\right)^5 + {}^5 C_5 \left(\frac{1}{2}\right)^5$$

$$= \frac{16}{2^5} = \frac{1}{2}$$

18. Let p and q be two positive number such that  $p + q = 2$  and  $p^4 + q^4 = 272$ . Then p and q are roots of the equation :

- (1)  $x^2 - 2x + 2 = 0$                       (2)  $x^2 - 2x + 8 = 0$   
 (3)  $x^2 - 2x + 136 = 0$                       (4)  $x^2 - 2x + 16 = 0$

**Ans. (4)**

Sol.  $(p^2 + q^2)^2 - 2p^2q^2 = 272$

$$((p + q)^2 - 2pq)^2 - 2p^2q^2 = 272$$

$$16 + 16pq + 2p^2q^2 = 272$$

$$(pq)^2 - 8pq - 128 = 0$$

$$pq = \frac{8 \pm 24}{2} = 16, -8$$

$$pq = 16$$

Now

$$x^2 - (p + q)x + pq = 0$$

$$x^2 - 2x + 16 = 0$$

19. If  $e^{(\cos^2 x + \cos^4 x + \cos^6 x + \dots) \log_e 2}$  satisfies the equation  $t^2 - 9t + 8 = 0$ , then the value of

$$\frac{2 \sin x}{\sin x + \sqrt{3} \cos x} \left( 0 < x < \frac{\pi}{2} \right) \text{ is :}$$

- (1)  $\frac{3}{2}$                       (2)  $2\sqrt{3}$                       (3)  $\frac{1}{2}$                       (4)  $\sqrt{3}$

Ans. (3)

Sol.  $e^{(\cos^2 x + \cos^4 x + \dots) \ln 2} = 2^{\cos^2 x + \cos^4 x + \dots}$

$$= 2^{\cot^2 x}$$

$$t^2 - 9t + 8 = 0 \Rightarrow t = 1, 8$$

$$\Rightarrow 2^{\cot^2 x} = 1, 8 \Rightarrow \cot^2 x = 0, 3$$

$$0 < x < \frac{\pi}{2} \Rightarrow \cot x = \sqrt{3}$$

$$\Rightarrow \frac{2 \sin x}{\sin x + \sqrt{3} \cos x} = \frac{2}{1 + \sqrt{3} \cot x} = \frac{2}{4} = \frac{1}{2}$$

20. The system of linear equations

$$3x - 2y - kz = 10$$

$$2x - 4y - 2z = 6$$

$$x + 2y - z = 5m$$

is inconsistent if :

- (1)  $k = 3, m = \frac{4}{5}$                       (2)  $k \neq 3, m \in \mathbb{R}$   
 (3)  $k \neq 3, m \neq \frac{4}{5}$                       (4)  $k = 3, m \neq \frac{4}{5}$

Ans. (4)

Sol.  $\Delta = \begin{vmatrix} 3 & -2 & -k \\ 1 & -4 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 0$

$$3(4 + 4) + 2(-2 + 2) - k(4 + 4) = 0$$

$$\Rightarrow k = 3$$

$$\Delta_x = \begin{vmatrix} 10 & -2 & -3 \\ 6 & -4 & -2 \\ 5m & 2 & -1 \end{vmatrix} \neq 0$$

$$10(4 + 4) + 2(-6 + 10m) - 3(12 + 20m) \neq 0$$

$$80 - 12 + 20m - 36 - 60m \neq 0$$

$$40m \neq 32 \Rightarrow m \neq \frac{4}{5}$$

$$\Delta_y = \begin{vmatrix} 3 & 10 & -3 \\ 2 & 6 & -2 \\ 1 & 5m & -1 \end{vmatrix} \neq 0$$

$$3(-6 + 10m) - 10(-2 + 2) - 3(10m - 6) \neq 0$$

$$-18 + 30m - 30m + 18 \neq 0 \Rightarrow 0$$

$$\Delta_z = \begin{vmatrix} 3 & -2 & 10 \\ 2 & -4 & 6 \\ 1 & 2 & 5m \end{vmatrix} \neq 0$$

$$3(-20m - 12) + 2(10m - 6) + 10(4 + 4) - 40m + 32 \neq 0 \Rightarrow m \neq \frac{4}{5}$$

### Section – B

1. Let  $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$ , where  $\alpha \in \mathbb{R}$ . Suppose  $Q = [q_{ij}]$  is a matrix satisfying  $PQ = kI_3$  for some

non-zero  $k \in \mathbb{R}$ . If  $q_{23} = -\frac{k}{8}$  and  $|Q| = \frac{k^2}{2}$ , then  $\alpha^2 + k^2$  is equal to \_\_\_\_\_

**Ans. 17**

Sol. As  $PQ = kI \Rightarrow Q = kP^{-1}I$

$$\text{now } Q = \frac{k}{|P|} (\text{adj}P) I \Rightarrow Q = \frac{k}{(20+12\alpha)} \begin{bmatrix} - & - & - \\ - & - & (-3\alpha-4) \\ - & - & - \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore q_{23} = \frac{-k}{8} \Rightarrow \frac{k}{(20+12\alpha)} (-3\alpha-4) = \frac{-k}{8} \Rightarrow 2(3\alpha+4) = 5+3\alpha$$

$$3\alpha = -3 \Rightarrow \alpha = -1$$

$$\text{also } |Q| = \frac{k^3 |I|}{|P|} \Rightarrow \frac{k^2}{2} = \frac{k^3}{(20+12\alpha)}$$

$$(20+12\alpha) = 2k \Rightarrow 8 = 2k \Rightarrow k = 4$$

2. Let  $B_i (i=1, 2, 3)$  be three independent events in a sample space. The probability that only  $B_1$  occur is  $\alpha$ , only  $B_2$  occurs is  $\beta$  and only  $B_3$  occurs is  $\gamma$ . Let  $p$  be the probability that none of the events  $B_i$  occurs and these 4 probabilities satisfy the equations  $(\alpha - 2\beta)p = \alpha\beta$  and  $(\beta - 3\gamma)p = 2\beta\gamma$  (All the probabilities are assumed to lie in the interval  $(0, 1)$ ). Then  $\frac{P(B_1)}{P(B_3)}$  is equal to \_\_\_\_\_

**Ans. 6**

Sol. Let  $x, y, z$  be probability of  $B_1, B_2, B_3$  respectively

$$\Rightarrow x(1-y)(1-z) = \alpha$$

$$\Rightarrow y(1-x)(1-z) = \beta$$

$$\Rightarrow z(1-x)(1-y) = \gamma$$

$$\Rightarrow (1-x)(1-y)(1-z) = p$$

$$(\alpha - 2\beta)p = \alpha\beta$$

$$(x(1-y)(1-z) - 2y(1-x)(1-z))(1-x)(1-y)(1-z) = \alpha\beta$$

$$x - xy - 2y + 2xy = \alpha\beta$$

$$x = 2y \quad \dots(1)$$

Similarly  $(\beta - 3\gamma)p = 2\beta\gamma$

$$\Rightarrow y = 3z \quad \dots(2)$$

From (1) & (2)

$$x = 6z$$

Now

$$\frac{x}{z} = 6$$

3. The minimum value of  $\alpha$  for which the equation  $\frac{4}{\sin x} + \frac{1}{1 - \sin x} = \alpha$  has at least one solution in

$\left(0, \frac{\pi}{2}\right)$  is \_\_\_\_\_

**Ans. 9**

Sol.  $f(x) = \frac{4}{\sin x} + \frac{1}{1 - \sin x}$

Let  $\sin x = t \quad \because x \in \left(0, \frac{\pi}{2}\right) \Rightarrow 0 < t < 1$

$$f(t) = \frac{4}{t} + \frac{1}{1-t}$$

$$f'(t) = \frac{-4}{t^2} + \frac{1}{(1-t)^2}$$

$$= \frac{t^2 - 4(1-t)^2}{t^2(1-t)^2}$$

$$= \frac{(t-2(1-t))(t+2(1-t))}{t^2(1-t)^2}$$

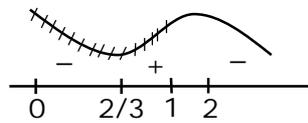
$$= \frac{(3t-2)(2-t)}{t^2(1-t)^2}$$

$$f_{\min} \text{ at } t = \frac{2}{3}$$

$$a_{\min} = f\left(\frac{2}{3}\right) = \frac{4}{\frac{2}{3}} + \frac{1}{1-\frac{2}{3}}$$

$$= 6 + 3$$

$$= 9$$



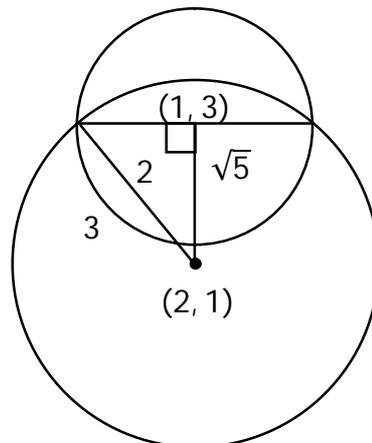
4. If one of the diameters of the circle  $x^2 + y^2 - 2x - 6y + 6 = 0$  is a chord of another circle 'C' whose center is at  $(2,1)$ , then its radius is \_\_\_\_\_

**Ans. 3**

distance between  $(1, 3)$  and  $(2, 1)$  is  $\sqrt{5}$

$$\therefore (\sqrt{5})^2 + (2)^2 = r^2$$

$$\Rightarrow r = 3$$



5.  $\lim_{x \rightarrow \infty} \tan \left\{ \sum_{r=1}^n \tan^{-1} \left( \frac{1}{1+r+r^2} \right) \right\}$  is equal to \_\_\_\_\_

Ans. 1

Sol. 
$$\begin{aligned} & \tan \left( \lim_{n \rightarrow \infty} \sum_{r=1}^n [\tan^{-1}(r+1) - \tan^{-1}(r)] \right) \\ &= \tan \left( \lim_{n \rightarrow \infty} \left( \tan^{-1}(n+1) - \frac{\pi}{4} \right) \right) \\ &= \tan \left( \frac{\pi}{4} \right) = 1 \end{aligned}$$

6. If  $\int_{-a}^a (|x| + |x-2|) dx = 22$ , ( $a > 2$ ) and  $[x]$  denotes the greatest integer  $\leq x$ , then  $\int_a^{-a} (x + [x]) dx$  is equal to \_\_\_\_\_

Ans. 3

Sol. 
$$\begin{aligned} & \int_{-a}^0 (-2x+2) dx + \int_0^2 (x+2-x) dx + \int_2^a (2x-2) dx = 22 \\ & x^2 - 2x \Big|_0^a + 2x \Big|_0^2 + x^2 - 2x \Big|_2^a = 22 \\ & a^2 + 2a + 4 + a^2 - 2a - (4 - 4) = 22 \\ & 2a^2 = 18 \Rightarrow a = 3 \\ & \int_3^{-3} (x + [x]) dx = - \left( \int_{-3}^3 (x + [x]) dx \right) = - \left( \int_{-3}^3 [x] dx \right) \\ &= -(-3 - 2 - 1 + 0 + 1 + 2) = 3 \end{aligned}$$

7. Let three vectors  $\vec{a}, \vec{b}$  and  $\vec{c}$  be such that  $\vec{c}$  is coplanar with  $\vec{a}$  and  $\vec{b}$ ,  $\vec{a} \cdot \vec{c} = 7$  and  $\vec{b}$  is perpendicular to  $\vec{c}$ , where  $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$  and  $\vec{b} = 2\hat{i} + \hat{k}$ , then the value of  $2|\vec{a} + \vec{b} + \vec{c}|^2$  is \_\_\_\_\_

Ans. 75

Sol. 
$$\begin{aligned} \vec{c} &= \lambda (\vec{b} \times (\vec{a} \times \vec{b})) \\ &= \lambda ((\vec{b} \cdot \vec{b}) \vec{a} - (\vec{b} \cdot \vec{a}) \vec{b}) \\ &= \lambda (5(-\hat{i} + \hat{j} + \hat{k}) + 2\hat{i} + \hat{k}) \end{aligned}$$

$$= \lambda(-3\hat{i} + 5\hat{j} + 6\hat{k})$$

$$\vec{c} \cdot \vec{a} = 7 \Rightarrow 3\lambda + 5\lambda + 6\lambda = 7$$

$$\lambda = \frac{1}{2}$$

$$\therefore 2 \left| \left( \frac{-3}{2} - 1 + 2 \right) \hat{i} + \left( \frac{5}{2} + 1 \right) \hat{j} + (3 + 1 + 1) \hat{k} \right|^2$$

$$= 2 \left( \frac{1}{4} + \frac{49}{4} + 25 \right) = 25 + 50 = 75$$

8. Let  $A = \{n \in \mathbb{N} : n \text{ is a 3-digit number}\}$

$$B = \{9k + 2 : k \in \mathbb{N}\}$$

and  $C = \{9k + \ell : k \in \mathbb{N}\}$  for some  $\ell$  ( $0 < \ell < 9$ )

If the sum of all the elements of the set  $A \cap (B \cup C)$  is  $274 \times 400$ , then  $\ell$  is equal to \_\_\_

**Ans. 5**

Sol. 3 digit number of the form  $9K + 2$  are  $\{101, 109, \dots, 992\}$

$$\Rightarrow \text{Sum equal to } \frac{100}{2} (1093) = s_1 = 54650$$

$$274 \times 400 = s_1 + s_2$$

$$274 \times 400 = \frac{100}{2} [101 + 992] + s_2$$

$$274 \times 400 = 50 \times 1093 + s_2$$

$$s_2 = 109600 - 54650$$

$$s_2 = 54950$$

$$s_2 = 54950 = \frac{100}{2} [(99 + \ell) + (990 + \ell)]$$

$$1099 = 2\ell + 1089$$

$$\ell = 5$$

9. If the least and the largest real values of  $\alpha$ , for which the equation  $z + \alpha|z-1| + 2i = 0$  ( $z \in \mathbb{C}$  and  $i = \sqrt{-1}$ ) has a solution, are  $p$  and  $q$  respectively; then  $4(p^2 + q^2)$  is equal to \_\_\_\_

**Ans. 10**

Sol.  $x + iy + \alpha\sqrt{(x-1)^2 + y^2} + 2i = 0$

$$\therefore y + 2 = 0 \text{ and } x + \alpha\sqrt{(x-1)^2 + y^2} = 0$$

$$y = -2 \text{ \& } x^2 = \alpha^2(x^2 - 2x + 1 + 4)$$

$$\alpha^2 = \frac{x^2}{x^2 - 2x + 5} \Rightarrow x^2(\alpha^2 - 1) - 2x\alpha^2 + 5\alpha^2 = 0$$

$$x \in \mathbb{R} \Rightarrow D \geq 0$$

$$4\alpha^4 - 4(\alpha^2 - 1)5\alpha^2 \geq 0$$

$$\alpha^2 [4\alpha^2 - 2\alpha^2 + 20] \geq 0$$

$$\alpha^2 [-16\alpha^2 + 20] \geq 0$$

$$\alpha^2 \left[ \alpha^2 - \frac{5}{4} \right] \leq 0$$

$$0 \leq \alpha^2 \leq \frac{5}{4}$$

$$\therefore \alpha^2 \in \left[ 0, \frac{5}{4} \right]$$

$$\therefore \alpha \in \left[ -\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2} \right]$$

$$\text{then } 4[(q)^2 + (p)^2] = 4 \left[ \frac{5}{4} + \frac{5}{4} \right] = 10$$

10. Let  $M$  be any  $3 \times 3$  matrix with entries from the set  $\{0, 1, 2\}$ . The maximum number of such matrices, for which the sum of diagonal elements of  $M^T M$  is seven, is \_\_\_\_

**Ans. 540**

Sol.  $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

$$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$$

Case I : Seven (1's) and two (0's)

$${}^9C_2 = 36$$

Case II : One (2) and three (1's) and five (0's)

$$\frac{9!}{5!3!} = 504$$

$\therefore$  Total = 540